1 Exam 1

2 Math 3351, Fall 2207

Instructions: Do the following problems using either a calculator or Maple. If you use Maple, you can turn in your answers as a Maple worksheet.

Show all your work! If you need to show some hand computation and are using Maple, you can either type it into the worksheet using

Maple's text facilities (see Maple help), or you can just leave some blank space in the worksheet and write it in by hand. If there is any question about what you can or can't use Maple commands to do, be sure to ask me.

```
> total_points := 50 + 40 + 40 + 50 + 40 + 40 + 50 + 40 + 80;
```

$$total\_points := 430$$

2.1 Problem 1. [50 pts]

Consider the matrix

```
> A := Matrix(3, 3, {(1, 1) = 1, (1, 2) = -2, (1, 3) = 2, (2, 1)
= 1, (2, 2) = 3, (2, 3) = 1, (3, 1) = 2, (3, 2) = 0, (3, 3) = 5});
```

$$A := \begin{bmatrix} 1 & -2 & 2 \\ 1 & 3 & 1 \\ 2 & 0 & 5 \end{bmatrix}$$

Part A. Show how to find the cofactors andA_{32}.

Part B. Show how to compute the determinant of$A$ by expanding along a row or column.

Part C. Find the adjoint matrix of$A$ by computing 2 by 2 determinants.

Part D. Use the information above to findA^{-1} .

2.2 Problem 2. [40 pts]

Find the determinant of A by the method of elimination.

```
> A :=Matrix(4, 4, {(1, 1) = 2, (1, 2) = 2, (1, 3) = 13, (1, 4)
= 8, (2, 1) = 5, (2, 2) = -3, (2, 3) = 16, (2, 4) = 16, (3, 1) =
2, (3, 2) = -2, (3, 3) = 4, (3, 4) = 6, (4, 1) = 3, (4, 2) = -3,
(4, 3) = 7, (4, 4) = 10});
```

$$A := \begin{bmatrix} 2 & 2 & 13 & 8 \\ 5 & -3 & 16 & 16 \\ 2 & -2 & 4 & 6 \\ 3 & -3 & 7 & 10 \end{bmatrix}$$

2.3 Problem 3. [40 pts]

Find the solution of the system by Cramer's rule.

```
> {3*x+5*y=7, x-2*y=5};
```

$$\{3\,x + 5\,y = 7, x - 2\,y = 5\}$$

2.4 Problem 4. [50 pts]

Consider the matrix

```
> A := Matrix([[6, 1, 0, -4, -1, 31, -15], [1, 3, 1, 8, -6, 32,
12], [49, -4, -2, -63, 14, 146, -179], [9, -2, -2, -19, 10, 4, -49],
[-5, 1, 1, 10, -5, -4, 26], [26, -1, 0, -28, 1, 97, -81], [40, -4,
-2, -54, 15, 108, -154]]);
```

$$A := \begin{bmatrix} 6 & 1 & 0 & -4 & -1 & 31 & -15 \\ 1 & 3 & 1 & 8 & -6 & 32 & 12 \\ 49 & -4 & -2 & -63 & 14 & 146 & -179 \\ 9 & -2 & -2 & -19 & 10 & 4 & -49 \\ -5 & 1 & 1 & 10 & -5 & -4 & 26 \\ 26 & -1 & 0 & -28 & 1 & 97 & -81 \\ 40 & -4 & -2 & -54 & 15 & 108 & -154 \end{bmatrix}$$

A. Find a basis of the nullspace of$A$.

B. Find a basis of the rowspace of$A$.

C. Find a basis of the columnspace of$A$

D. What is the rank of$A$?

2.5 Problem 5. [40 pts]

In each part, determine if the given vectors are linearly dependent or linearly independent. If they are dependent, find a linear relation between them,

i.e., find constantsc_i (not all zero) so that$\mathit{delayDotProduct}\left(c_1, v_1, \mathit{true}\right) + \mathit{delayDotProduct}\left(c_1, v_2, \mathit{true}\right)$ -

Part A.

```
> (v[1], v[2], v[3], v[4]) := Vector(5, {(1) = 5, (2) = -1, (3)
= 1, (4) = -1, (5) = 1}), Vector(5, {(1) = -7, (2) = -3, (3) = 13,
(4) = -2, (5) = 5}), Vector(5, {(1) = 73, (2) = 19, (3) = 2, (4)
= -24, (5) = -24}), Vector(5, {(1) = -154, (2) = -35, (3) = 52,
(4) = 28, (5) = 51});
```

$$v_1, v_2, v_3, v_4 := \begin{bmatrix} 5 \\ -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -7 \\ -3 \\ 13 \\ -2 \\ 5 \end{bmatrix}, \begin{bmatrix} 73 \\ 19 \\ 2 \\ -24 \\ -24 \end{bmatrix}, \begin{bmatrix} -154 \\ -35 \\ 52 \\ 28 \\ 51 \end{bmatrix}$$

Part B.

```
> (v[1], v[2], v[3], v[4]):=Vector(5, {(1) = 1, (2) = 2, (3) =
8, (4) = -3, (5) = 5}), Vector(5, {(1) = -1, (2) = -15, (3) = 7,
(4) = 2, (5) = 3}), Vector(5, {(1) = -3, (2) = -32, (3) = 6, (4)
= 7, (5) = 1}), Vector(5, {(1) = 4, (2) = 21, (3) = 17, (4) = -11,
(5) = 12});
```

$$v_1, v_2, v_3, v_4 := \begin{bmatrix} 1 \\ 2 \\ 8 \\ -3 \\ 5 \end{bmatrix}, \begin{bmatrix} -1 \\ -15 \\ 7 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -3 \\ -32 \\ 6 \\ 7 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ 21 \\ 17 \\ -11 \\ 12 \end{bmatrix}$$

2.6 Problem 6. [40 pts]

In each part, determine if the given vectors span$\mathbb{R}^4$. If so, extract as basis of$real^4$ from the given vectors. Part A. .

```
> Vector(4, {(1) = -2, (2) = -6, (3) = 17, (4) = 2}), Vector(4,
{(1) = 2, (2) = 5, (3) = 2, (4) = 2}), Vector(4, {(1) = 6, (2) =
17, (3) = -32, (4) = -2}), Vector(4, {(1) = 16, (2) = 43, (3) =
-41, (4) = 4}), Vector(4, {(1) = 0, (2) = 1, (3) = 3, (4) = 1}),
Vector(4, {(1) = 1, (2) = 4, (3) = -10, (4) = -1}), Vector(4, {(1)
= -3, (2) = 6, (3) = -27, (4) = -4});
```

$$\begin{bmatrix} -2 \\ -6 \\ 17 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 6 \\ 17 \\ -32 \\ -2 \end{bmatrix}, \begin{bmatrix} 16 \\ 43 \\ -41 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \\ -10 \\ -1 \end{bmatrix}, \begin{bmatrix} -3 \\ 6 \\ -27 \\ -4 \end{bmatrix}$$

Part B.

```
> Vector(4, {(1) = -2, (2) = -28, (3) = 23, (4) = -10}), Vector(4,
{(1) = 9, (2) = 129, (3) = -102, (4) = 34}), Vector(4, {(1) = 1,
(2) = 15, (3) = -11, (4) = 1}), Vector(4, {(1) = 25, (2) = 361,
(3) = -282, (4) = 83}), Vector(4, {(1) = 10, (2) = 150, (3) = -110,
(4) = 11}), Vector(4, {(1) = 1, (2) = 13, (3) = -12, (4) = 10});
```

$$\begin{bmatrix} -2 \\ -28 \\ 23 \\ -10 \end{bmatrix}, \begin{bmatrix} 9 \\ 129 \\ -102 \\ 34 \end{bmatrix}, \begin{bmatrix} 1 \\ 15 \\ -11 \\ 1 \end{bmatrix}, \begin{bmatrix} 25 \\ 361 \\ -282 \\ 83 \end{bmatrix}, \begin{bmatrix} 10 \\ 150 \\ -110 \\ 11 \end{bmatrix}, \begin{bmatrix} 1 \\ 13 \\ -12 \\ 10 \end{bmatrix}$$

2.7 Problem 7. [50 pts]

Let$S$ be the subspace of$real^4$ spanned by the vectors

```
> (v[1], v[2], v[3], v[4], v[5]):=Vector(4, {(1) = -43, (2) = -2,
(3) = -3, (4) = 35}), Vector(4, {(1) = -24, (2) = 0, (3) = -1, (4)
= 20}), Vector(4, {(1) = 62, (2) = 4, (3) = 5, (4) = -50}), Vector(4,
{(1) = 34, (2) = 1, (3) = 2, (4) = -28}), Vector(4, {(1) = 19, (2)
= 2, (3) = 2, (4) = -15});
```

$$v_1, v_2, v_3, v_4, v_5 := \begin{bmatrix} -43 \\ -2 \\ -3 \\ 35 \end{bmatrix}, \begin{bmatrix} -24 \\ 0 \\ -1 \\ 20 \end{bmatrix}, \begin{bmatrix} 62 \\ 4 \\ 5 \\ -50 \end{bmatrix}, \begin{bmatrix} 34 \\ 1 \\ 2 \\ -28 \end{bmatrix}, \begin{bmatrix} 19 \\ 2 \\ 2 \\ -15 \end{bmatrix}$$

Part A. Find a basis of$S$. What is the dimension of$S$?

Part B. Determine if each of the followin vectors is in$S$ and, if so, express it as a linear combination of the basis vectors you found in Part A.

```
> (w[1],w[2]):=Vector(4, {(1) = -150, (2) = 1, (3) = -7, (4) =
125}), Vector(4, {(1) = 64, (2) = -1, (3) = 2, (4) = -54});
```

$$w_1, w_2 := \begin{bmatrix} -150 \\ 1 \\ -7 \\ 125 \end{bmatrix}, \begin{bmatrix} 64 \\ -1 \\ 2 \\ -54 \end{bmatrix}$$

2.8 Problem 8. [40 pts]

Part A. LetS_1 be the subspace of$real^4$ spanned by the vectors

```
> (v1, v2, v3) := Vector(4, {(1) = 4, (2) = -6, (3) = 4, (4) =
9}), Vector(4, {(1) = 0, (2) = 2, (3) = -1, (4) = -4}), Vector(4,
{(1) = 7, (2) = -3, (3) = 3, (4) = 0});
```

$$v1,\ v2,\ v3 := \begin{bmatrix} 4 \\ -6 \\ 4 \\ 9 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -1 \\ -4 \end{bmatrix}, \begin{bmatrix} 7 \\ -3 \\ 3 \\ 0 \end{bmatrix}$$

Let S_2 be the subpace spanned by the vectors

```
> w1 := Vector(4, {(1) = 43, (2) = -29, (3) = 24, (4) = 22}); w2
:= Vector(4, {(1) = -3, (2) = 5, (3) = -3, (4) = -7}); w3 := Vector(4,
{(1) = 27, (2) = -45, (3) = 29, (4) = 69});
```

$$w1 := \begin{bmatrix} 43 \\ -29 \\ 24 \\ 22 \end{bmatrix}$$

$$w2 := \begin{bmatrix} -3 \\ 5 \\ -3 \\ -7 \end{bmatrix}$$

$$w3 := \begin{bmatrix} 27 \\ -45 \\ 29 \\ 69 \end{bmatrix}$$

Is S_1 a subset of S_2? Is S_2 a subset of S_1 ? Are S_1 and S_2 equal?

Part B. Let S_3 be the subspace spanned by the vectors

```
> (u[1], u[2], u[3]) := Vector(4, {(1) = -17, (2) = 7, (3) = -7,
(4) = 1}), Vector(4, {(1) = 63, (2) = -65, (3) = 47, (4) = 79}),
Vector(4, {(1) = 179, (2) = -269, (3) = 179, (4) = 403});
```

$$u_1,\ u_2,\ u_3 := \begin{bmatrix} -17 \\ 7 \\ -7 \\ 1 \end{bmatrix}, \begin{bmatrix} 63 \\ -65 \\ 47 \\ 79 \end{bmatrix}, \begin{bmatrix} 179 \\ -269 \\ 179 \\ 403 \end{bmatrix}$$

Is S_1 a subset of S_3? Is S_3 a subset of S_1 ? Are S_1 and S_3 equal?

2.9 Problem 9. [80 pts]

In each part, Find the characteristic polynomial of A.

Find the eigenvalues of A.

Find a basis for each of the eigenspaces of A.

Determine if A is diagonalizable. If so, find an invertible matrix P and a diagonal matrix C so that $P^{-1}AP = C$. Part A.

```
> A := Matrix([[-4, -2, 9], [-14, -1, 18], [-2, -2, 7]]);
```

$$A := \begin{bmatrix} -4 & -2 & 9 \\ -14 & -1 & 18 \\ -2 & -2 & 7 \end{bmatrix}$$

Part B.

```
> A := Matrix([[21, 21, -3, -32], [12, 16, -3, -18], [40, 46, -7,
-62], [16, 18, -3, -24]]);
```

$$A := \begin{bmatrix} 21 & 21 & -3 & -32 \\ 12 & 16 & -3 & -18 \\ 40 & 46 & -7 & -62 \\ 16 & 18 & -3 & -24 \end{bmatrix}$$

Part C.

```
> A := Matrix([[1, 1, 1, 0], [-18, -86, -15, 48], [21, 101, 18,
-56], [-27, -130, -22, 73]]);
```

$$A := \begin{bmatrix} 1 & 1 & 1 & 0 \\ -18 & -86 & -15 & 48 \\ 21 & 101 & 18 & -56 \\ -27 & -130 & -22 & 73 \end{bmatrix}$$

Part D.

```
> a := Matrix([[-758, 444, -1041, 125], [-1145, 668, -1574, 191],
[-83, 47, -115, 15], [-1248, 727, -1717, 209]]);
```

$$a := \begin{bmatrix} -758 & 444 & -1041 & 125 \\ -1145 & 668 & -1574 & 191 \\ -83 & 47 & -115 & 15 \\ -1248 & 727 & -1717 & 209 \end{bmatrix}$$