EXAM

Exam 3

Math 2350, Fall 2007

Nov 30, 2007

- This is a Takehome Exam, due Thursday, December 6.
- You may discuss the problems with other people, including other members of the class, but write up your own solutions.
- Write all of your answers on separate sheets of paper. You can keep the exam questions.
- You **must** show enough work to justify your answers. Unless otherwise instructed, give exact answers, not approximations (e.g., $\sqrt{2}$, not 1.414).
- Unless otherwise stated, you can use a calculator to evaluate the integrals. Always write out the integral to be found and indicate in your answers where you used the calculator.
- This exam has 7 problems. There are **300 points total**.

40 pts.

Problem 1. Find the absolute max and min of the function $f(x, y) = x^2 + y^2 - 4y$ on the rectangle given by $-1 \leq x \leq 1$ and $0 \leq y \leq 3$.

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40 pts.

Problem 2. Us Lagrange Multipliers to find the absolute maximum and minimum of the function $f(x, y, z) = xy + z^2$ on the spherical surface $x^2 + y^2 + z^2 = 2$.

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60 pts.

Problem 3. Let R be the region in three dimensional space bounded by the planes $y = 0$, $z = y$ and the cylinder $z = 1 - x^2$.

A. Set up an iterated integral for finding the volume of R, where the first integration is with respect to y.

B. Set up an iterated integral for finding the volume of R, where the first integration is with respect to z.

C. Evaluate one of these integrals.

In this problem, work the integral by hand.

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40 pts.

Problem 4. Let D be the solid bounded below by the paraboloid $z = x^2 + y^2$ and above by the plane $z = 1$. Find $\bar{z}$, the z-coordinate of the centroid of D.

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40 pts.

Problem 5. Let D be the solid hemisphere bounded above by the sphere $x^2 + y^2 + z^2 = a^2$ and below by the xy-plane. Find the volume of D, $\bar{z}$ (the z-coordinate of the centroid) and the moment of inertia of D for rotation about the z-axis.

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40 pts.

Problem 6. Let the surface S be the upper hemisphere of the sphere $x^2 + y^2 + z^2 = a^2$ (i.e., the part of the sphere in the region $z \geq 0$). Find $\bar{z}$, the z-coordinate of the centroid of S and the moment of inertia of S for rotation about the z-axis. [These are surface integrals.]

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40 pts.

Problem 7. Let the surface S be the part of the cone $z = \sqrt{x^2 + y^2}$ that lies in the region $0 \leq z \leq 2$. Let $\mathbf{F}(x, y, z)$ be the vector field

$$\mathbf{F}(x, y, z) = xz\mathbf{i} + yz\mathbf{j}.$$

Calculate the flux integral

$$\iint_S \mathbf{F} \cdot \mathbf{n}\, dS,$$

where $\mathbf{n}$ is the outward pointing unit normal on S.
