

Problem 1. In each part, find the general solution of the given differential equation.

A.

$$\frac{dy}{dx} = x + xy^2.$$

B.

$$\frac{dy}{dx} = \frac{x^2 + 2y^2}{xy}$$

C.

$$(1.1) \quad \frac{dy}{dx} - \tan(x)y = \frac{1}{\cos^3(x)}$$

D.

$$(1.3) \quad \frac{dy}{dx} + y = y^3 e^{3x}$$

E.

$$\frac{dy}{dx} = x(2x + y)^2 - 2.$$

Problem 2. A tank contains 100 gallons of water. Initially there are 20 pounds of salt dissolved in the tank. Water flows into the tank at a rate of 10 gallons per minute. Each gallon of incoming water contains 2 pounds of dissolved salt. The mixture in the tank is kept practically uniform by stirring. Water flows out of the tank at a rate of 10 gallons per minute.

What is the limiting value of the amount of salt in the tank as $t \rightarrow \infty$? At what time does the amount of salt in the tank reach 98% of its limiting value? At what time does it reach 99% of the limiting value? (Give approximations of the answers accurate to two decimal places.)

Problem 3. An object with a mass of 100 kilograms is dropped from a height of 2000 meters. The force of air resistance on the object is proportional to the velocity, and the terminal velocity of the object is 50 meters per second. How long does it take for the object to reach the ground?

Problem 4. Solve the differential equation

$$(x - y + 2) dx + (x + y - 4) dy = 0.$$

Problem 5. Consider the differential equation

(*)
$$y dx + 3x dy = 0.$$

1. Find an integrating factor for (*) that is a function of y alone and use it to solve the equation.
 2. Find an integrating factor for (*) that is a function of x alone and use it to solve the equation.
 3. Are your answers for the first two parts of the problem compatible? Explain.
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EXAM

Practice Questions for Exam #1

Math 3350, Spring 2006

Feb.12, 2005

- Write all of your answers on separate sheets of paper. You can keep the exam questions when you leave. You may leave when finished.
- You **must** show enough work to justify your answers. Unless otherwise instructed, give exact answers, not approximations (e.g., $\sqrt{2}$, not 1.414).
- This exam has 5 problems. There are **180 points total**.

Good luck!