
EXAM

Exam #3

Math 3350, Spring 2006

April 22, 2005

- This is a Take Home Exam. It is due on Tuesday, May 2, by 5 p.m.
- You **must** show enough work to justify your answers. Unless otherwise instructed, give exact answers, not approximations (e.g., $\sqrt{2}$, not 1.414).
- You may discuss the problems with other people, but write up the solutions by yourself.
- You can use a calculator, particularly to do integrals and partial fraction decompositions. Indicate what you are going to compute using the calculator and what the result is. If there is any doubt about what is legal to do with a calculator, ask me.
- This exam has 6 problems. There are **230 points total**.

Good luck!

40 pts.

Problem 1. In each part, find the Inverse Laplace Transform.

A. In this part, work out the partial fractions decomposition by hand,

$$\frac{1}{s^2(s-1)}.$$

B. In this part, you can do the partial fractions decomposition by machine,

$$\frac{3s^3 - 2s^2 + 2s + 2}{(s^2 + 4)(s - 1)^2}.$$

40 pts.

Problem 2. Find the Laplace transform of the function

$$f(t) = \begin{cases} t^2, & 0 \leq t < 1, \\ t, & 1 \leq t < 2 \\ 1, & 2 < t < \infty \end{cases}$$

20 pts.

Problem 3. Find the Inverse Laplace Transform of the function

$$\frac{1}{(s-1)} + e^{-s} \frac{1}{s^2+4} + e^{-2s} \frac{1}{(s^2+1)}$$

40 pts.

Problem 4. In each part, solve the initial value problem **using Laplace transforms**. (No credit for using a different method.)

A.

$$y'' - 2y' + y = te^t, \quad y(0) = 1, \quad y'(0) = 0.$$

B.

$$y'' + 4y = u(t-1) + tu(t-2), \quad y(0) = 0, \quad y'(0) = 1.$$

40 pts.

Problem 5.

A. Compute the convolution $t * t^2$ directly from the definition of convolution.

B. Compute the convolution $t^3 * \sin(2t)$, using Laplace transforms.

50 pts.

Problem 6.

- A. Let $f(x)$ be the function of period $2L = 4$ which is given on the interval $(-2, 2)$ by $f(x) = x$. Find the Fourier Series of $f(x)$.
- B. Find the Fourier sine and cosine series of the function $f(x) = x$ on the interval $[0, 2]$.
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