70 pts.

Problem 1. Consider the matrix

$$A = \begin{bmatrix} -1 & 3 & 11 & 0 & 5 & 2 & 18 \\ -1 & 5 & 17 & 1 & 10 & 2 & 22 \\ 1 & -4 & -14 & -1 & -8 & -2 & -19 \\ -1 & 1 & 5 & 0 & 1 & 1 & 7 \\ 3 & 4 & 6 & 2 & 13 & 0 & 11 \\ 1 & 3 & 7 & 0 & 7 & 1 & 15 \end{bmatrix}.$$

The RREF of A is the matrix

$$R = \begin{bmatrix} 1 & 0 & -2 & 0 & 1 & 0 & 1 \\ 0 & 1 & 3 & 0 & 2 & 0 & 3 \\ 0 & 0 & 0 & 1 & 1 & 0 & -2 \\ 0 & 0 & 0 & 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

A. Find a basis for the nullspace of A.

B. Find a basis for the rowspace of A.

C. Find a basis for the columnspace of A.

D. What is the rank of A?

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40 pts.

Problem 2.

A. Consider the vectors

$$v_1 = \begin{bmatrix} 3 \\ 1 \\ 1 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 3 \\ 2 \\ 1 \\ 1 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 2 \\ 1 \\ 1 \\ -1 \end{bmatrix}$$

in $\mathbb{R}^4$. Determine if these vectors are linearly independent or linearly dependent. If they are dependent, find scalars c_1, c_2 and c_3, not all zero, so that $c_1v_1 + c_2v_2 + c_3v_3 = 0$.

B. Consider the vectors

$$v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 1 \\ 2 \\ 2 \\ 1 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 4 \\ 4 \\ 1 \end{bmatrix}$$

in $\mathbb{R}^4$. Determine if these vectors are linearly independent or linearly dependent. If they are dependent, find scalars c_1, c_2 and c_3, not all zero, so that $c_1v_1 + c_2v_2 + c_3v_3 = 0$.

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55 pts.

Problem 3. Let $S = \operatorname{span}(v_1, v_2, v_3, v_4, v_5)$ be the subspace of $\mathbb{R}^5$ spanned by the vectors

$$v_1 = \begin{bmatrix} 2 \\ -1 \\ 1 \\ -3 \\ 5 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 2 \\ 0 \\ 0 \\ -3 \\ 5 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 2 \\ 1 \\ -1 \\ -3 \\ 5 \end{bmatrix}, \quad v_4 = \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ -2 \end{bmatrix}, \quad v_5 = \begin{bmatrix} 6 \\ -2 \\ 1 \\ -6 \\ 12 \end{bmatrix}.$$

A. Find a basis for S. What is the dimension of S?

B. Consider the vectors

$$u = \begin{bmatrix} 13 \\ -3 \\ 1 \\ -13 \\ 25 \end{bmatrix}, \quad w = \begin{bmatrix} 0 \\ 1 \\ 2 \\ -9 \\ 9 \end{bmatrix}.$$

Determine if each of these vector is in S and, if so, express it as a linear combination of the basis vectors of S you found in the first part.

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60 pts.

Problem 4. Consider the vectors

$$u_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \qquad u_2 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

in $\mathbb{R}^2$.

A. Show that $\mathcal{U} = \begin{bmatrix} u_1 & u_2 \end{bmatrix}$ is an ordered basis of $\mathbb{R}^2$. Find the change of basis matrices $S_{\mathcal{E}\mathcal{U}}$ and $S_{\mathcal{U}\mathcal{E}}$, where $\mathcal{E}$ is the standard basis of $\mathbb{R}^2$.

B. Let $v \in \mathbb{R}^2$ be the vector

$$v = \begin{bmatrix} 2 \\ 3 \end{bmatrix}.$$

Find $[v]_{\mathcal{U}}$, the coordinate vector of v with respect to the basis $\mathcal{U}$.

C. Let $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation whose matrix with respect to the standard basis is

$$[T]_{\mathcal{E}\mathcal{E}} = \begin{bmatrix} 9 & -14 \\ 7 & -12 \end{bmatrix}.$$

Find $[T]_{\mathcal{U}\mathcal{U}}$, the matrix of T with respect to the basis $\mathcal{U}$.

D. Find $[T(v)]_{\mathcal{U}}$, the coordinates of $T(v)$ with respect to $\mathcal{U}$, where v is the vector in part B.

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40 pts.

Problem 5. Let

$$A = \begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix}.$$

Find the characteristic polynomial and the eigenvalues of A. (Do not find any eigenvectors.)

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60 pts.

Problem 6. In each part, you are given a matrix A and its eigenvalues. Find a basis for each of the eigenspaces of A and determine if A is diagonalizable. If so, find a diagonal matrix D and an invertible matrix P so that $P^{-1}AP = D$.

A. The matrix is

$$A = \begin{bmatrix} 8 & -6 & -6 \\ 6 & -4 & -6 \\ 3 & -3 & -1 \end{bmatrix}$$

and the eigenvalues are -1 and 2.

B. The matrix is

$$A = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$$

and the eigenvalue is 2.

EXAM

Exam #2

Math 2360, Spring 2006

April 4, 2006

- Write all of your answers on separate sheets of paper. You can keep the exam questions when you leave. You may leave when finished.
- You **must** show enough work to justify your answers. Unless otherwise instructed, give exact answers, not approximations (e.g., $\sqrt{2}$, not 1.414).
- This exam has 6 problems. There are **325 points total**.

Good luck!