50 pts.

Problem 1. In each part you are given the augmented matrix of a system of linear equations, with the coefficent matrix in reduced row echelon form. Determine if the system is consistent and, if it is consistent, find all solutions.

A.

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & -3 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

B.

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -2 \\ 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

C.

$$\left[\begin{array}{ccccc|c} 1 & 0 & -2 & 0 & 2 & 2 \\ 0 & 1 & -1 & 0 & 3 & -1 \\ 0 & 0 & 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 0 & 2 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

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45 pts.

Problem 2. Consider row operations on matrices with 3 rows.

Recall that for each row operation there is a corresponding elementary matrix E so that EA is the same as the matrix obtained by applying the row operation to A.

A. Consider the row operation $R_2 \leftrightarrow R_3$.

   i.) Find the corresponding elementary matrix E.

   ii.) Find the inverse row operation.

   iii.) Find the elementary matrix that corresponds to the inverse row operation.

B. Consider the row operation $R_3 \leftarrow 3R_3$.

   i.) Find the corresponding elementary matrix E.

   ii.) Find the inverse row operation.

   iii.) Find the elementary matrix that corresponds to the inverse row operation.

C. Consider the row operation $R_2 \leftarrow R_2 - 3R_1$

   i.) Find the corresponding elementary matrix E.

   ii.) Find the inverse row operation.

   iii.) Find the elementary matrix that corresponds to the inverse row operation.

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50 pts.

Problem 3. The matrix

$$A = \begin{bmatrix} 0 & 2 \\ 2 & 4 \end{bmatrix}$$

is invertible.

A. Find the inverse of A using row operations. You can use the calculator to perform the row operations, but show which row operations you are performing and the resulting matrices.

B. Using the information from the last part, express A as a product of elementary matrices.

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70 pts.

Problem 4. Consider the matrix

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & -1 & 2 \\ 2 & 1 & 0 \end{bmatrix}$$

A. Find the cofactors A_{12} and A_{33}.

B. Compute $\det(A)$, using the cofactor expansion along a selected row or column.

C. Find the adjoint matrix $\operatorname{adj}(A)$ from its definition in terms of cofactors.

D. **Use the information above** to find A^{-1}.

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40 pts.

Problem 5. Find the following determinant by the method of elimination, i.e., by using row operations and keeping track of the effect of the row operations on the determinant. Sorry, no credit for finding it by another method.

$$\begin{vmatrix} 4 & 4 & 0 \\ 6 & -1 & 1 \\ 2 & 3 & 2 \end{vmatrix}$$

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40 pts.

Problem 6. Show how to use **Cramer's rule** to solve the following system.

$$\begin{aligned} 2x &+ 3y = 8 \\ 2x &+ 6y = 0 \end{aligned}$$

EXAM

Exam #1

Math 2360, Spring 2006

Feb 16, 2005

- Write all of your answers on separate sheets of paper. You can keep the exam questions when you leave. You may leave when finished.
- You **must** show enough work to justify your answers. Unless otherwise instructed, give exact answers, not approximations (e.g., $\sqrt{2}$, not 1.414).
- This exam has 6 problems. There are **295 points total**.

Good luck!