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## **PROBLEM SET**

Problems on Variation of Parameters

Math 3322, Fall 2006

November 1, 2006

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## ANSWERS



**Problem 1.**

In each part, solve find the general solution of the given differential equation by the method of variation of parameters. Some of these can also be done by the method of undetermined coefficients, but don't do it that way.

1.  $y'' + y = \sec^2(x)$ .

*Answer:*  $y = -1 + \ln|\sec(x) + \tan(x)| \sin(x) + C_1 \cos(x) + C_2 \sin(x)$

2.  $y'' + \omega^2 y = \sin(\omega t)$ .

*Answer:*  $y = -t \cos(\omega t)/(2\omega) + C_1 \cos(\omega t) + C_2 \sin(\omega t)$

3.  $y'' - 4y' + 4y = e^{2x}/x^2$ .

*Answer:*  $y = \ln|x|e^{2x} + C_1 e^{2x} + C_2 x e^{2x}$

4.  $y'' + y' - 2y = x e^x$ .

*Answer:*  $y = (3x^2 - 2x)e^x/18 + C_1 e^x + C_2 e^{-2x}$

5.  $y'' + y' - 2y = x e^{-x}$ .

*Answer:*  $y = (1/4 - x/2)e^{-x} + C_1 e^x + C_2 e^{-2x}$

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**Problem 2.**

In this problem, you may assume that the general solution of the equation

$$\frac{d^2y}{dx^2} + \frac{1}{x} \frac{dy}{dx} - \frac{1}{x^2}y = 0, \quad x > 0,$$

is given by

$$y = C_1x + C_2\frac{1}{x}$$

so the basic solutions are  $y_1 = x$  and  $y_2 = 1/x$ . Use these basic solutions and the method of variation of parameters to find the general solution of the following differential equations.

1.

$$\frac{d^2y}{dx^2} + \frac{1}{x} \frac{dy}{dx} - \frac{1}{x^2}y = x^2.$$

*Answer:*

$$y = \frac{x^4}{15} + C_1x + C_2\frac{1}{x}$$

2.

$$\frac{d^2y}{dx^2} + \frac{1}{x} \frac{dy}{dx} - \frac{1}{x^2}y = \frac{1}{x}$$

*Answer:*

$$y = \frac{1}{2}x \ln(x) + C_1x + C_2\frac{1}{x}$$

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