
EXAM

Exam 3

Math 3322, Fall 2006

November 29, 2006

- This is a takehome exam. You may work with other students, but write up your own answers.
- You **must** show enough work to justify your answers. Unless otherwise instructed, give exact answers, not approximations (e.g., $\sqrt{2}$, not 1.414).
- This exam has 7 problems. There are **340 points total**.

Good luck!

40 pts. **Problem 1.** In each part, find the general solution of the differential equation.

A. $D^2(D - 2)^2(D + 1)^3y = 0$

B. $(D^2 + 6D + 13)^2y = 0$

40 pts. **Problem 2.** In each part, use the method of undetermined coefficients to find the general solution of the differential equation.

A. $(D^2 + 4)y = x^2$

B. $(D^2 - 3D + 2)y = xe^{2x}$.

40 pts. **Problem 3.** Use the method of variation of parameters to find the general solution of the equation

$$(D^2 - 4D + 4)y = \frac{e^{2x}}{x^3}.$$

60 pts. **Problem 4.** In each part, find the partial fraction decomposition of the given rational function using hand computation. Show your work!

A.
$$\frac{4s^2 - 7s + 4}{s(s - 1)(s - 2)}$$

B.
$$\frac{3s^2 - 3s + 2}{s^2(s - 1)}$$

C.
$$\frac{2s^3 + 5s^2 + 2}{s^2(s^2 + 1)}$$

40 pts. **Problem 5.** In each part, find the Laplace transform of the given function.

A. $f(t) = 2t^3 + 3t^2 + t + 2$.

B. $f(t) = e^{5t} \cos(3t)$.

C. $f(t) = t^2 e^{3t} + 2e^{-2t}$.

D. $f(t) = t \sin(2t)$.

60 pts.

Problem 6. In each part, find the inverse Laplace transform of the given function. You can use a calculator for the partial fractions calculations.

A.

$$F(s) = \frac{1}{s^2(s-1)(s-2)}$$

B.

$$F(s) = \frac{2s^3 + 3s + 5}{(s-1)^2(s^2 + 4)}$$

C.

$$F(s) = \frac{s+1}{s^2 - 4s + 8}$$

60 pts.

Problem 7. In each part, use Laplace transforms to solve the differential equation. You can use a calculator to do the partial fraction decompositions.

A. $y'' - y' - 2y = e^{-t}$, with initial conditions $y(0) = 1$ and $y'(0) = 1$.

B. $y'' + y = \sin(t)$, with initial conditions $y(0) = 0$ and $y'(0) = 1$.
