EXAM

Exam #3

Math 2350, Spring 2005

April 25, 2005

- This is a take-home exam, due by 5 p.m., Monday, May 2.
- Write all of your answers on separate sheets of paper. You can keep the exam questions.
- You **must** show enough work to justify your answers. Unless otherwise instructed, give exact answers, not approximations (e.g., $\sqrt{2}$, not 1.414).
- This exam has 9 problems. There are **360 points total**.

Good luck!

Instructions: You can work together, but after figuring out how to do the problem, write up your own solution. I should not be seeing identical text.

Unless otherwise instructed, you can use a symbolic calculator to do the integrals. However, do one one-dimensional integral at a time and indicate clearly where you have used the calculator.

---

40 pts.

Problem 1. Us Lagrange Multipliers to find the absolute maximum and minimum of the function $f(x, y, z) = xy + z$ on the spherical surface $x^2 + y^2 + z^2 = 9$.

---

40 pts.

Problem 2. Let R be the region in the xy-plane bounded by $y = x^2$ and $y = 2x$. Find the integral

$$\iint_R x \, dx \, dy.$$

In this problem, work the integrals by hand and show the steps.

---

40 pts.

Problem 3. In each part, sketch the region of integration and find an equivalent iterated integral with the order of integration reversed.

A.

$$\int_0^1 \int_{\sqrt{x}}^1 f(x, y) \, dy \, dx.$$

B.

$$\int_1^2 \int_{-\sqrt{2-y}}^{\sqrt{2-y}} f(x, y) \, dx \, dy.$$

---

40 pts.

Problem 4. Let R be the region in the first quadrant enclosed by one leaf of the four leaf rose $r = \sin(2\theta)$. Find $\bar{x}$, the x-coordinate of the centroid of R. What is $\bar{y}$?

---

40 pts.

Problem 5. Let D be the region that in three-dimensional space bounded below by $z = 0$, above by $z = y$ and laterally by the plane $y = 1$ and the cylinder $y = x^2$.

A. Find an iterated integral for finding the volume of D where the first integration is with respect to z.

B. Find an iterated integral for finding the volume of D where the first integration is with respect to x.

C. Find an iterated integral for finding the volume of D where the first integration is with respect to y.

D. Evaluate one of these integrals by hand.

---

40 pts.

Problem 6. Let D be the region in three-dimensional space bounded above by the paraboloid $z = 6 - x^2 - y^2$ and below by the cone $z = \sqrt{x^2 + y^2}$. Find the volume of D, and the moment of inertia for rotation about the z-axis.

---

40 pts.

Problem 7. Let D be the region in three-dimensional space bounded above by the sphere $x^2+y^2+z^2 = a^2$ and below by the cone $z = \sqrt{x^2 + y^2}$. Use spherical coordinates to find the volume of D, $\bar{z}$ (the z-coordinate of the centroid) and the moment of inertia for rotation about the z-axis.

---

40 pts.

Problem 8. Let D be the spherical cap bounded above by the sphere $x^2 + y^2 + z^2 = a^2$ and below by the plane $z = h$, where $0 < h < a$. Use a triple integral to find the volume of D.

---

40 pts.

Problem 9. Let R be the region in the xy-plane bounded below by the x-axis and above by the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

Find $\bar{y}$, the y-coordinate of the centroid of R.

Start by making the change of variables $x = au$, $y = bv$.
