
EXAM

Take-home Exam
Exam #3

Math 3350-01, Second Summer Session 2004

July 30, 2004

- Write all of your answers on separate sheets of paper. You can keep the exam questions.
- You **must** show enough work to justify your answers. Unless otherwise instructed, give exact answers, not approximations (e.g., $\sqrt{2}$, not 1.414).
- This exam has 7 problems. There are **240 points total**.
- You may discuss the exam with other students, but write up the answers by yourself.
- **Due Monday, August 2.**

Good luck!

40 pts.

Problem 1. In each part, find the Inverse Laplace Transform.

A. In this part, work out the partial fractions decomposition by hand,

$$\frac{1}{s^2(s-1)}.$$

B. In this part, you can do the partial fractions decomposition by machine,

$$\frac{3s^3 - 2s^2 + 2s + 2}{(s^2 + 4)(s - 1)^2}.$$

40 pts.

Problem 2. Find the Laplace transform of the function

$$f(t) = \begin{cases} t, & 0 \leq t < 1, \\ t^2, & 1 \leq t < 2 \\ 0, & 2 < t < \infty \end{cases}$$

20 pts.

Problem 3. Find the Inverse Laplace Transform of the function

$$\frac{1}{s-1} + e^{-s} \frac{1}{s^2+4} + e^{-2s} \frac{1}{(s+2)^2}.$$

40 pts.

Problem 4. In each part, solve the initial value problem **using Laplace transforms**. (No credit for using a different method.)

A.

$$y'' - 4y' + 4y = te^{2t}, \quad y(0) = 1, \quad y'(0) = 0.$$

B.

$$y'' + 4y = \delta(t-1) + tu(t-2), \quad y(0) = 0, \quad y'(0) = 1.$$

60 pts.

Problem 5. In the following problems, use formulas (1) and (6) from section 5.4.

A. Find the Laplace transform of $f(t) = t^2 \cos(2t)$.

B. Find the Laplace transform of

$$f(t) = \frac{e^{2t} - e^t}{t}.$$

You may assume the fact that $\lim_{t \rightarrow 0^+} f(t)$ exists.

C. Find the inverse Laplace transform of the function

$$G(s) = \frac{1}{2} \ln\left(\frac{s^2}{s^2 + 1}\right).$$

20 pts.

Problem 6. Find the convolution $t * t^3$ directly from the definition.

20 pts.

Problem 7. Find the convolution $\cos(2t) * \sin(3t)$ **using Laplace transforms.**
