
EXAM

Practice Questions for Exam #3

Math 3350, Spring 2004

April 16, 2004

- Write all of your answers on separate sheets of paper. You can keep the exam questions when you leave. You may leave when finished.
- You **must** show enough work to justify your answers. Unless otherwise instructed, give exact answers, not approximations (e.g., $\sqrt{2}$, not 1.414).
- You may use the book, your notes, and the handouts given in class
- This exam has 8 problems.

Good luck!

Problem 1. Find the general solution **by the method of undetermined coefficients**:

$$(D^2 + 4)y = x \sin(2x).$$

Problem 2. Find the Laplace Transform of the following function:

$$f(t) = \begin{cases} 0, & 0 < t < 1 \\ t^2 + 2t, & 1 < t < 2 \\ 1, & 2 < t < \infty. \end{cases}$$

Problem 3. Find the Inverse Laplace Transform of the following function:

$$F(s) = \frac{1}{s+1} + e^{-s} \frac{1}{(s+2)^2} + e^{-3s} \frac{1}{s^2+4}$$

Problem 4. Solve the following initial value problem, **using Laplace Transforms**:

$$y'' - 2y' + y = t^2 u(t-1) + \delta(t-2), \quad y(0) = 0, \quad y'(0) = 1.$$

Problem 5. In the following problems, use formulas (1) and (6) from section 5.4.

1. Find the Laplace transform of $f(t) = te^{2t} \cos(t)$.
2. Find the Laplace transform of $f(t) = (e^{2t} - 1)/t$.
3. Find the inverse Laplace transform of the function

$$G(s) = \ln\left(\frac{s^2+1}{s^2+4}\right)$$

Problem 6. Find the convolution $t^2 * t$ directly from the definition.

Problem 7. Find the convolution $\cos(2t) * \sin(t)$ **using Laplace transforms**.

Problem 8. Express the solution of the following initial value problem using a convolution integral.

$$y'' - 2y' + y = r(t), \quad y(0) = 0, \quad y'(0) = 1.$$
