60 pts.

Problem 1. In each part, find the general solution of the given differential equation.

A.

$$\frac{dy}{dx} = \frac{x^2 + 2y^2}{xy}, \qquad u = y/x$$

B.

$$\frac{dy}{dx} - \tan(x)y = \frac{1}{\cos^3(x)} \tag{1.1}$$

C.

$$\frac{dy}{dx} + y = y^3 e^{3x}, \qquad \text{(Bernoulli equation)} \tag{1.3}$$

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60 pts.

Problem 2. A tank contains 100 gallons of water. Initially there are 20 pounds of salt dissolved in the tank. Water flows into the tank at a rate of 10 gallons per minute. Each gallon of incoming water contains 2 pounds of dissolved salt. The mixture in the tank is kept practically uniform by stirring. Water flows out of the tank at a rate of 10 gallons per minute.

What is the limiting value of the amount of salt in the tank as $t \to \infty$? At what time does the amount of salt in the tank reach 98% of its limiting value? At what time does it reach 99% of the limiting value? (Give approximations of the answers accurate to two decimal places.)

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60 pts.

Problem 3. Consider the differential equation

$$y\,dx + 3x\,dy = 0. \tag{$*$}$$

1. Find an integrating factor for $(*)$ that is a function of y alone and use it to solve the equation.
2. Find an integrating factor for $(*)$ that is a function of x alone and use it to solve the equation.
3. Are your answers for the first two parts of the problem compatible? Explain.

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60 pts.

Problem 4. In each part, solve the initial value problem.

A.

$$y'' + 2y' - 3y = 0, \quad y(0) = 1, \quad y'(0) = 1.$$

B.

$$y'' + 4y' + 4y = 0, \quad y(0) = 2, \quad y'(0) = 1.$$

C.

$$y'' - 6y' + 34y = 0, \quad y(0) = 0, \quad y'(0) = 2$$

EXAM

Practice Questions for Exam #1

Math 3350, Spring 2004

Feb.19, 2004

- Write all of your answers on separate sheets of paper. You can keep the exam questions when you leave. You may leave when finished.
- You **must** show enough work to justify your answers. Unless otherwise instructed, give exact answers, not approximations (e.g., $\sqrt{2}$, not 1.414).
- This exam has 4 problems. There are **240 points total**.

Good luck!