EXAM

Exam #3

Math 3350, Spring 2004

April 22, 2004

- This is a **Take-home Exam**. It is **due on Tuesday, April 27**.
- You may look at the Textbook, your notes, and any material given out in class.. You may use a calculator. You may discuss the problems with classmates, but write up your own solutions, don't just copy somebody!
- Write all of your answers on separate sheets of paper. You can keep the exam questions.
- You **must** show enough work to justify your answers. Unless otherwise instructed, give exact answers, not approximations (e.g., $\sqrt{2}$, not 1.414).
- This exam has 7 problems. There are **280 points total**.

Good luck!

40 pts.

Problem 1. Find the general solution **by the method of undetermined coefficients**:

$$(D^2 + 4)y = x^2 \cos(2x).$$

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40 pts.

Problem 2. Find the Laplace Transform of the following function:

$$f(t) = \begin{cases} t, & 0 < t < 1 \\ t^2 + 5t, & 1 < t < 2 \\ 0, & 2 < t < \infty. \end{cases}$$

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40 pts.

Problem 3. Find the Inverse Laplace Transform of the following function:

$$F(s) = \frac{1}{s} + e^{-s}\frac{1}{(s-1)(s-2)} + e^{-5s}\frac{1}{s^2+4}.$$

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40 pts.

Problem 4. Solve the following initial value problem, **using Laplace Transforms**:

$$y'' - 5y' + 6y = u(t-1)(2t+1), \quad y(0) = 1, \quad y'(0) = 0.$$

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40 pts.

Problem 5. In the following problems, use formulas (1) and (6) from section 5.4.

A. Find the Laplace transform of $f(t) = t^2 \sin(2t)$.

B. Find the Laplace transform of

$$f(t) = \frac{\cos(t) - \cos(2t)}{t}.$$

You may assume the fact that $\lim_{t \to 0^+} f(t)$ exists.

C. Find the inverse Laplace transform of the function

$$G(s) = \ln\left(\frac{s-1}{s+1}\right).$$

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40 pts.

Problem 6. Find the convolution $t * e^{2t}$ directly from the definition.

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40 pts.

Problem 7. Find the convolution $e^t * \cos(2t)$ **using Laplace transforms**.
