```
>
```

This worksheet will compare the books method of doing undetermined coefficients with the shifting rule method, in a couple of examples.

Example 1

```
> deqn := diff(y(x),x,x)-4*diff(y(x),x)+3*y(x) =
  x^2*exp(5*x)*cos(2*x);
```

$$deqn := \left(\frac{d^2}{dx^2} y(x)\right) - 4\left(\frac{d}{dx} y(x)\right) + 3\, y(x) = x^2\, \mathbf{e}^{(5\,x)} \cos(2\,x)$$

```
> charpoly := lambda^2 -4*lambda + 3;
```

$$charpoly := \lambda^2 - 4\,\lambda + 3$$

```
> factor(charpoly);
```

$$(\lambda - 1)\,(\lambda - 3)$$

```
>
```

Book's Method.

```
> yt :=
  (A*x^2+B*x+C)*exp(5*x)*cos(2*x)+(E*x^2+F*x+G)*exp(5*x)*sin(2*x
```

$$yt := (A\,x^2 + B\,x + C)\,\mathbf{e}^{(5\,x)} \cos(2\,x) + (E\,x^2 + F\,x + G)\,\mathbf{e}^{(5\,x)} \sin(2\,x)$$

```
> dyt := diff(yt,x);
```

$$\begin{aligned} dyt := {} & (2\,A\,x + B)\,\mathbf{e}^{(5\,x)} \cos(2\,x) + 5\,(A\,x^2 + B\,x + C)\,\mathbf{e}^{(5\,x)} \cos(2\,x) \\ & - 2\,(A\,x^2 + B\,x + C)\,\mathbf{e}^{(5\,x)} \sin(2\,x) + (2\,E\,x + F)\,\mathbf{e}^{(5\,x)} \sin(2\,x) \\ & + 5\,(E\,x^2 + F\,x + G)\,\mathbf{e}^{(5\,x)} \sin(2\,x) + 2\,(E\,x^2 + F\,x + G)\,\mathbf{e}^{(5\,x)} \cos(2\,x) \end{aligned}$$

```
> d2yt := diff(dyt,x);
```

$$\begin{aligned} d2yt := {} & 2\,A\,\mathbf{e}^{(5\,x)} \cos(2\,x) + 10\,(2\,A\,x + B)\,\mathbf{e}^{(5\,x)} \cos(2\,x) \\ & - 4\,(2\,A\,x + B)\,\mathbf{e}^{(5\,x)} \sin(2\,x) + 21\,(A\,x^2 + B\,x + C)\,\mathbf{e}^{(5\,x)} \cos(2\,x) \\ & - 20\,(A\,x^2 + B\,x + C)\,\mathbf{e}^{(5\,x)} \sin(2\,x) + 2\,E\,\mathbf{e}^{(5\,x)} \sin(2\,x) \end{aligned}$$

$$+ 10\,(2\,E\,x + F)\,\mathbf{e}^{(5\,x)}\,\sin(2\,x) + 4\,(2\,E\,x + F)\,\mathbf{e}^{(5\,x)}\,\cos(2\,x)$$
$$+ 21\,(E\,x^2 + F\,x + G)\,\mathbf{e}^{(5\,x)}\,\sin(2\,x) + 20\,(E\,x^2 + F\,x + G)\,\mathbf{e}^{(5\,x)}\,\cos(2\,x)$$

```
> lh := d2yt-4*dyt + 3*yt;
```

$$:= 2\,A\,\mathbf{e}^{(5\,x)}\,\cos(2\,x) + 6\,(2\,A\,x + B)\,\mathbf{e}^{(5\,x)}\,\cos(2\,x)$$
$$- 4\,(2\,A\,x + B)\,\mathbf{e}^{(5\,x)}\,\sin(2\,x) + 4\,(A\,x^2 + B\,x + C)\,\mathbf{e}^{(5\,x)}\,\cos(2\,x)$$
$$- 12\,(A\,x^2 + B\,x + C)\,\mathbf{e}^{(5\,x)}\,\sin(2\,x) + 2\,E\,\mathbf{e}^{(5\,x)}\,\sin(2\,x)$$
$$+ 6\,(2\,E\,x + F)\,\mathbf{e}^{(5\,x)}\,\sin(2\,x) + 4\,(2\,E\,x + F)\,\mathbf{e}^{(5\,x)}\,\cos(2\,x)$$
$$+ 4\,(E\,x^2 + F\,x + G)\,\mathbf{e}^{(5\,x)}\,\sin(2\,x) + 12\,(E\,x^2 + F\,x + G)\,\mathbf{e}^{(5\,x)}\,\cos(2\,x)$$

```
> f := lh - x^2*exp(5*x)*cos(2*x);
```

$$:= 2\,A\,\mathbf{e}^{(5\,x)}\,\cos(2\,x) + 6\,(2\,A\,x + B)\,\mathbf{e}^{(5\,x)}\,\cos(2\,x)$$
$$- 4\,(2\,A\,x + B)\,\mathbf{e}^{(5\,x)}\,\sin(2\,x) + 4\,(A\,x^2 + B\,x + C)\,\mathbf{e}^{(5\,x)}\,\cos(2\,x)$$
$$- 12\,(A\,x^2 + B\,x + C)\,\mathbf{e}^{(5\,x)}\,\sin(2\,x) + 2\,E\,\mathbf{e}^{(5\,x)}\,\sin(2\,x)$$
$$+ 6\,(2\,E\,x + F)\,\mathbf{e}^{(5\,x)}\,\sin(2\,x) + 4\,(2\,E\,x + F)\,\mathbf{e}^{(5\,x)}\,\cos(2\,x)$$
$$+ 4\,(E\,x^2 + F\,x + G)\,\mathbf{e}^{(5\,x)}\,\sin(2\,x) + 12\,(E\,x^2 + F\,x + G)\,\mathbf{e}^{(5\,x)}\,\cos(2\,x)$$
$$- x^2\,\mathbf{e}^{(5\,x)}\,\cos(2\,x)$$

```
>
```

We want to choose the coefficients so this is zero. So computer stuff to find the equations:

```
> f1 := subs(exp(5*x)=e, cos(2*x)=c, sin(2*x)=s, f);
```

$$:= 2\,A\,e\,c + 6\,(2\,A\,x + B)\,e\,c - 4\,(2\,A\,x + B)\,e\,s + 4\,(A\,x^2 + B\,x + C)\,e\,c$$
$$- 12\,(A\,x^2 + B\,x + C)\,e\,s + 2\,E\,e\,s + 6\,(2\,E\,x + F)\,e\,s + 4\,(2\,E\,x + F)\,e\,c$$
$$+ 4\,(E\,x^2 + F\,x + G)\,e\,s + 12\,(E\,x^2 + F\,x + G)\,e\,c - x^2\,e\,c$$

```
> f1 := f1/e;
```

$$:= \frac{1}{e}\,(2\,A\,e\,c + 6\,(2\,A\,x + B)\,e\,c - 4\,(2\,A\,x + B)\,e\,s + 4\,(A\,x^2 + B\,x + C)\,e\,c$$

$$- 12 (A x^2 + B x + C) e s + 2 E e s + 6 (2 E x + F) e s + 4 (2 E x + F) e c + 4 (E x^2 + F x + G) e s + 12 (E x^2 + F x + G) e c - x^2 e c)$$

```
> f2 := expand(f1);
```

$$2 := 2 A c + 12 c A x + 6 c B - 8 s A x - 4 s B + 4 c A x^2 + 4 c B x + 4 c C - 12 s A x^2 - 12 s B x - 12 s C + 2 E s + 12 s E x + 6 s F + 8 c E x + 4 c F + 4 s E x^2 + 4 s F x + 4 s G + 12 c E x^2 + 12 c F x + 12 c G - x^2 c$$

```
> f3 := collect(f2, [c,s]);
```

$$3 := (6 B + 4 B x + 4 C + 12 A x + 8 E x + 4 F + 2 A + 4 A x^2 + 12 E x^2 + 12 F x + 12 G - x^2) c + (6 F + 4 G - 12 C - 8 A x - 4 B - 12 B x + 2 E - 12 A x^2 + 4 F x + 12 E x + 4 E x^2) s$$

```
> p1 := coeff(f3, c);
```

$$p1 := 6 B + 4 B x + 4 C + 12 A x + 8 E x + 4 F + 2 A + 4 A x^2 + 12 E x^2 + 12 F x + 12 G - x^2$$

```
> p2 := coeff(f3, s);
```

$$:= 6 F + 4 G - 12 C - 8 A x - 4 B - 12 B x + 2 E - 12 A x^2 + 4 F x + 12 E x + 4 E x^2$$

```
> p1 := collect(p1,x);
```

$$1 := (4 A + 12 E - 1) x^2 + (12 A + 8 E + 4 B + 12 F) x + 6 B + 4 F + 4 C + 2 A + 12 G$$

```
> p2 := collect(p2,x);
```

$$p2 := (-12 A + 4 E) x^2 + (-8 A - 12 B + 4 F + 12 E) x + 6 F + 4 G - 12 C - 4 B + 2 E$$

```
> eqs := {};
```

$$eqs := \{\}$$

```
> for i from 0 to 2 do eqs := eqs union {coeff(p1,x,i)=0}; od;
```

$$eqs := \{6 B + 4 F + 4 C + 2 A + 12 G = 0\}$$

$$eqs := \{6 B + 4 F + 4 C + 2 A + 12 G = 0, 12 A + 8 E + 4 B + 12 F = 0\}$$

$$eqs := \{6 B + 4 F + 4 C + 2 A + 12 G = 0, 12 A + 8 E + 4 B + 12 F = 0,$$

$4A + 12E - 1 = 0\}$

```
> for i from 0 to 2 do eqs := eqs union {coeff(p2, x, i)=0}; od;
```

$$eqs := \{6B + 4F + 4C + 2A + 12G = 0, 12A + 8E + 4B + 12F = 0, 4A + 12E - 1 = 0, 6F + 4G - 12C - 4B + 2E = 0\}$$

$$eqs := \{6B + 4F + 4C + 2A + 12G = 0, 12A + 8E + 4B + 12F = 0, 4A + 12E - 1 = 0, 6F + 4G - 12C - 4B + 2E = 0, -8A - 12B + 4F + 12E = 0\}$$

$$eqs := \{6B + 4F + 4C + 2A + 12G = 0, 12A + 8E + 4B + 12F = 0, 4A + 12E - 1 = 0, 6F + 4G - 12C - 4B + 2E = 0, -8A - 12B + 4F + 12E = 0, -12A + 4E = 0\}$$

```
> eqs;
```

$$\{6B + 4F + 4C + 2A + 12G = 0, 12A + 8E + 4B + 12F = 0, 4A + 12E - 1 = 0, 6F + 4G - 12C - 4B + 2E = 0, -8A - 12B + 4F + 12E = 0, -12A + 4E = 0\}$$

```
> sols := solve(eqs, {A,B,C,E,F,G});
```

$$sols := \left\{ C = \frac{-133}{4000}, G = \frac{81}{4000}, F = \frac{-17}{200}, B = \frac{3}{100}, A = \frac{1}{40}, E = \frac{3}{40} \right\}$$

```
> yp := subs(sols, yt);
```

$$yp := \left(\frac{1}{40}x^2 + \frac{3}{100}x - \frac{133}{4000}\right) e^{(5x)} \cos(2x) + \left(\frac{3}{40}x^2 - \frac{17}{200}x + \frac{81}{4000}\right) e^{(5x)} \sin(2x)$$

```
> subs(y(x)=yp, deqn);
```

$$\left(\frac{d^2}{dx^2}\left(\left(\frac{1}{40}x^2 + \frac{3}{100}x - \frac{133}{4000}\right) e^{(5x)} \cos(2x) + \left(\frac{3}{40}x^2 - \frac{17}{200}x + \frac{81}{4000}\right) e^{(5x)} \sin(2x)\right)\right)$$

$$- 4\left(\frac{d}{dx}\left(\left(\frac{1}{40}x^2 + \frac{3}{100}x - \frac{133}{4000}\right)\mathbf{e}^{(5\,x)}\cos(2\,x)\right.\right.$$

$$\left.\left.+\left(\frac{3}{40}x^2 - \frac{17}{200}x + \frac{81}{4000}\right)\mathbf{e}^{(5\,x)}\sin(2\,x)\right)\right)$$

$$+ 3\left(\frac{1}{40}x^2 + \frac{3}{100}x - \frac{133}{4000}\right)\mathbf{e}^{(5\,x)}\cos(2\,x)$$

$$+ 3\left(\frac{3}{40}x^2 - \frac{17}{200}x + \frac{81}{4000}\right)\mathbf{e}^{(5\,x)}\sin(2\,x) = x^2\,\mathbf{e}^{(5\,x)}\cos(2\,x)$$

```
> eval(%);
```

$$\frac{1}{20}\mathbf{e}^{(5\,x)}\cos(2\,x) + 6\left(\frac{1}{20}x + \frac{3}{100}\right)\mathbf{e}^{(5\,x)}\cos(2\,x)$$

$$- 4\left(\frac{1}{20}x + \frac{3}{100}\right)\mathbf{e}^{(5\,x)}\sin(2\,x)$$

$$+ 4\left(\frac{1}{40}x^2 + \frac{3}{100}x - \frac{133}{4000}\right)\mathbf{e}^{(5\,x)}\cos(2\,x)$$

$$- 12\left(\frac{1}{40}x^2 + \frac{3}{100}x - \frac{133}{4000}\right)\mathbf{e}^{(5\,x)}\sin(2\,x) + \frac{3}{20}\mathbf{e}^{(5\,x)}\sin(2\,x)$$

$$+ 6\left(\frac{3}{20}x - \frac{17}{200}\right)\mathbf{e}^{(5\,x)}\sin(2\,x) + 4\left(\frac{3}{20}x - \frac{17}{200}\right)\mathbf{e}^{(5\,x)}\cos(2\,x)$$

$$+ 4\left(\frac{3}{40}x^2 - \frac{17}{200}x + \frac{81}{4000}\right)\mathbf{e}^{(5\,x)}\sin(2\,x)$$

$$+ 12\left(\frac{3}{40}x^2 - \frac{17}{200}x + \frac{81}{4000}\right)\mathbf{e}^{(5\,x)}\cos(2\,x) = x^2\,\mathbf{e}^{(5\,x)}\cos(2\,x)$$

```
> simplify(%);
```

$$x^2\,\mathbf{e}^{(5\,x)}\cos(2\,x) = x^2\,\mathbf{e}^{(5\,x)}\cos(2\,x)$$

```
>
```

Now try the shifting rule method.

```
>
```

```
> cmplxdeqn :=diff(y(x),x,x)-4*diff(y(x),x) +
  3*y(x)=exp((5+2*I)*x)*x^2;
```

$$cmplxdeqn := \left(\frac{d^2}{dx^2} y(x)\right) - 4\left(\frac{d}{dx} y(x)\right) + 3\, y(x) = \mathbf{e}^{((5+2\,I)\,x)}\, x^2$$

```
> subs(lambda = d+5+2*I, charpoly);
```

$$(d + (5 + 2\,I))^2 - 4\,d + (-17 - 8\,I)$$

```
> expand(%);
```

$$d^2 + 6\,d + 4\,I\,d + (4 + 12\,I)$$

```
> collect(%,d);
```

$$(4 + 12\,I) + d^2 + (6 + 4\,I)\,d$$

This operator, with d = D, is the operator we should apply to find $z = \exp(-(5 + 2\,I)\,x)\,y$.

```
> zt := A*x^2+B*x+C;
```

$$zt := A\,x^2 + B\,x + C$$

```
> diff(zt,x,x)+(6+4*I)*diff(zt,x)+(4+12*I)*zt;
```

$$2\,A + (6 + 4\,I)\,(2\,A\,x + B) + (4 + 12\,I)\,(A\,x^2 + B\,x + C)$$

```
> f:=% - x^2;
```

$$f := 2\,A + (6 + 4\,I)\,(2\,A\,x + B) + (4 + 12\,I)\,(A\,x^2 + B\,x + C) - x^2$$

```
> f1:=collect(f,x);
```

$$:= ((4 + 12\,I)\,A - 1)\,x^2 + ((12 + 8\,I)\,A + (4 + 12\,I)\,B)\,x + 2\,A + (6 + 4\,I)\,B + (4 + 12\,I)\,C$$

```
> eqset :={};
```

$$eqset := \{\}$$

```
> for i from 0 to 2 do eqset := eqset union {coeff(f1,x,i)=0};
  od;
```

$$eqset := \{2\,A + (6 + 4\,I)\,B + (4 + 12\,I)\,C = 0\}$$

$$set := \{2\,A + (6 + 4\,I)\,B + (4 + 12\,I)\,C = 0, (12 + 8\,I)\,A + (4 + 12\,I)\,B = 0\}$$

$set := \{2\,A + (6 + 4\,I)\,B + (4 + 12\,I)\,C = 0, (12 + 8\,I)\,A + (4 + 12\,I)\,B = 0, (4 + 12\,I)\,A - 1 = 0\}$

```
> eqset;
```

$$\{2\,A + (6 + 4\,I)\,B + (4 + 12\,I)\,C = 0, (12 + 8\,I)\,A + (4 + 12\,I)\,B = 0, (4 + 12\,I)\,A - 1 = 0\}$$

```
> sols := solve(eqset, {A,B,C});
```

$$sols := \left\{A = \frac{1}{40} - \frac{3}{40}\,I, B = \frac{3}{100} + \frac{17}{200}\,I, C = -\frac{133}{4000} - \frac{81}{4000}\,I\right\}$$

```
> zp := subs(sols, zt);
```

$$zp := \left(\frac{1}{40} - \frac{3}{40}\,I\right)x^2 + \left(\frac{3}{100} + \frac{17}{200}\,I\right)x + \left(-\frac{133}{4000} - \frac{81}{4000}\,I\right)$$

```
> yc := exp((5+2*I)*x)*zp;
```

$$:= \mathbf{e}^{((5 + 2\,I)\,x)}\left(\left(\frac{1}{40} - \frac{3}{40}\,I\right)x^2 + \left(\frac{3}{100} + \frac{17}{200}\,I\right)x + \left(-\frac{133}{4000} - \frac{81}{4000}\,I\right)\right)$$

```
> assume(x, real);
> evalc(yc);
```

$$\mathbf{e}^{(5\,x\sim)}\cos(2\,x\sim)\left(\frac{1}{40}x\sim^2 + \frac{3}{100}x\sim - \frac{133}{4000}\right) - \mathbf{e}^{(5\,x\sim)}\sin(2\,x\sim)\left(-\frac{3}{40}x\sim^2 + \frac{17}{200}x\sim - \frac{81}{4000}\right) + I\left(\mathbf{e}^{(5\,x\sim)}\sin(2\,x\sim)\left(\frac{1}{40}x\sim^2 + \frac{3}{100}x\sim - \frac{133}{4000}\right) + \mathbf{e}^{(5\,x\sim)}\cos(2\,x\sim)\left(-\frac{3}{40}x\sim^2 + \frac{17}{200}x\sim - \frac{81}{4000}\right)\right)$$

```
> yp := Re(yc);
```

$$yp := \frac{1}{40}\mathbf{e}^{(5\,x\sim)}\cos(2\,x\sim)\,x\sim^2 + \frac{3}{100}\mathbf{e}^{(5\,x\sim)}\cos(2\,x\sim)\,x\sim - \frac{133}{4000}\mathbf{e}^{(5\,x\sim)}\cos(2\,x\sim) + \frac{3}{40}\mathbf{e}^{(5\,x\sim)}\sin(2\,x\sim)\,x\sim^2 - \frac{17}{200}\mathbf{e}^{(5\,x\sim)}\sin(2\,x\sim)\,x\sim + \frac{81}{4000}\mathbf{e}^{(5\,x\sim)}\sin(2\,x\sim)$$

```
>
```

Second Example

```
> x := 'x';
```

$$x := x$$

```
> charpoly := (lambda - (2+3*I))*(lambda-(2-3*I));
```

$$charpoly := (\lambda + (-2 - 3\,I))\,(\lambda + (-2 + 3\,I))$$

```
> expand(charpoly);
```

$$\lambda^2 - 4\,\lambda + 13$$

```
> deqn := diff(y(x),x,x)-4*diff(y(x),x) + 13*y(x) =
  x^2*exp(2*x)*cos(3*x);
```

$$deqn := \left(\frac{d^2}{dx^2}\,y(x)\right) - 4\left(\frac{d}{dx}\,y(x)\right) + 13\,y(x) = x^2\,\mathbf{e}^{(2\,x)}\cos(3\,x)$$

```
>
```

Book's method.

```
> yt := (A*x^2+B*x+C)*exp(2*x)*cos(3*x)+
  (E*x^2+F*x+G)*exp(2*x)*sin(3*x);
```

$$yt := (A\,x^2 + B\,x + C)\,\mathbf{e}^{(2\,x)}\cos(3\,x) + (E\,x^2 + F\,x + G)\,\mathbf{e}^{(2\,x)}\sin(3\,x)$$

```
> frontend(expand,[yt]);
```

$$x)\ \cos(3\,x)\,A\,x^2 + x\,\mathbf{e}^{(2\,x)}\cos(3\,x)\,B + \mathbf{e}^{(2\,x)}\cos(3\,x)\,C + \mathbf{e}^{(2\,x)}\sin(3\,x)\,E\,x^2 + x\,\mathbf{e}^{(2\,x)}\sin(3\,x)\,F + \mathbf{e}^{(2\,x)}\sin(3\,x)\,G$$

```
> yt := x*yt;
```

$$t := x\left((A\,x^2 + B\,x + C)\,\mathbf{e}^{(2\,x)}\cos(3\,x) + (E\,x^2 + F\,x + G)\,\mathbf{e}^{(2\,x)}\sin(3\,x)\right)$$

```
> yt:=frontend(expand,[yt]);
```

$$yt := \mathbf{e}^{(2\,x)}\cos(3\,x)\,A\,x^3 + x^2\,\mathbf{e}^{(2\,x)}\cos(3\,x)\,B + x\,\mathbf{e}^{(2\,x)}\cos(3\,x)\,C + \mathbf{e}^{(2\,x)}\sin(3\,x)\,E\,x^3 + x^2\,\mathbf{e}^{(2\,x)}\sin(3\,x)\,F + x\,\mathbf{e}^{(2\,x)}\sin(3\,x)\,G$$

```
> dyt := diff(yt,x);
```

$$
\begin{aligned}
dyt := {}& 2\,\mathbf{e}^{(2\,x)} \cos(3\,x)\, A\, x^3 - 3\,\mathbf{e}^{(2\,x)} \sin(3\,x)\, A\, x^3 + 3\,\mathbf{e}^{(2\,x)} \cos(3\,x)\, A\, x^2 \\
&+ 2\,x\,\mathbf{e}^{(2\,x)} \cos(3\,x)\, B + 2\,x^2\,\mathbf{e}^{(2\,x)} \cos(3\,x)\, B - 3\,x^2\,\mathbf{e}^{(2\,x)} \sin(3\,x)\, B \\
&+ \mathbf{e}^{(2\,x)} \cos(3\,x)\, C + 2\,x\,\mathbf{e}^{(2\,x)} \cos(3\,x)\, C - 3\,x\,\mathbf{e}^{(2\,x)} \sin(3\,x)\, C \\
&+ 2\,\mathbf{e}^{(2\,x)} \sin(3\,x)\, E\, x^3 + 3\,\mathbf{e}^{(2\,x)} \cos(3\,x)\, E\, x^3 + 3\,\mathbf{e}^{(2\,x)} \sin(3\,x)\, E\, x^2 \\
&+ 2\,x\,\mathbf{e}^{(2\,x)} \sin(3\,x)\, F + 2\,x^2\,\mathbf{e}^{(2\,x)} \sin(3\,x)\, F + 3\,x^2\,\mathbf{e}^{(2\,x)} \cos(3\,x)\, F \\
&+ \mathbf{e}^{(2\,x)} \sin(3\,x)\, G + 2\,x\,\mathbf{e}^{(2\,x)} \sin(3\,x)\, G + 3\,x\,\mathbf{e}^{(2\,x)} \cos(3\,x)\, G
\end{aligned}
$$

```
> d2yt := diff(dyt,x);
```

$$
\begin{aligned}
2yt := {}& 2\,\mathbf{e}^{(2\,x)} \cos(3\,x)\, B + 4\,\mathbf{e}^{(2\,x)} \sin(3\,x)\, G + 2\,\mathbf{e}^{(2\,x)} \sin(3\,x)\, F \\
&+ 4\,\mathbf{e}^{(2\,x)} \cos(3\,x)\, C + 6\,\mathbf{e}^{(2\,x)} \cos(3\,x)\, A\, x + 6\,\mathbf{e}^{(2\,x)} \sin(3\,x)\, E\, x \\
&+ 12\,\mathbf{e}^{(2\,x)} \cos(3\,x)\, A\, x^2 + 8\,x\,\mathbf{e}^{(2\,x)} \cos(3\,x)\, B + 12\,\mathbf{e}^{(2\,x)} \sin(3\,x)\, E\, x^2 \\
&+ 8\,x\,\mathbf{e}^{(2\,x)} \sin(3\,x)\, F - 18\,\mathbf{e}^{(2\,x)} \sin(3\,x)\, A\, x^2 - 12\,x\,\mathbf{e}^{(2\,x)} \sin(3\,x)\, B \\
&+ 18\,\mathbf{e}^{(2\,x)} \cos(3\,x)\, E\, x^2 + 12\,x\,\mathbf{e}^{(2\,x)} \cos(3\,x)\, F - 5\,\mathbf{e}^{(2\,x)} \cos(3\,x)\, A\, x^3 \\
&- 5\,x^2\,\mathbf{e}^{(2\,x)} \cos(3\,x)\, B - 5\,x\,\mathbf{e}^{(2\,x)} \cos(3\,x)\, C - 5\,\mathbf{e}^{(2\,x)} \sin(3\,x)\, E\, x^3 \\
&- 5\,x^2\,\mathbf{e}^{(2\,x)} \sin(3\,x)\, F - 5\,x\,\mathbf{e}^{(2\,x)} \sin(3\,x)\, G - 12\,\mathbf{e}^{(2\,x)} \sin(3\,x)\, A\, x^3 \\
&- 12\,x^2\,\mathbf{e}^{(2\,x)} \sin(3\,x)\, B - 12\,x\,\mathbf{e}^{(2\,x)} \sin(3\,x)\, C + 12\,\mathbf{e}^{(2\,x)} \cos(3\,x)\, E\, x^3 \\
&- 6\,\mathbf{e}^{(2\,x)} \sin(3\,x)\, C + 6\,\mathbf{e}^{(2\,x)} \cos(3\,x)\, G + 12\,x^2\,\mathbf{e}^{(2\,x)} \cos(3\,x)\, F \\
&+ 12\,x\,\mathbf{e}^{(2\,x)} \cos(3\,x)\, G
\end{aligned}
$$

```
> lh := d2yt-4*dyt+13*yt;
```

$$
\begin{aligned}
lh := {}& 2\,\mathbf{e}^{(2\,x)} \cos(3\,x)\, B + 2\,\mathbf{e}^{(2\,x)} \sin(3\,x)\, F + 6\,\mathbf{e}^{(2\,x)} \cos(3\,x)\, A\, x \\
&+ 6\,\mathbf{e}^{(2\,x)} \sin(3\,x)\, E\, x - 18\,\mathbf{e}^{(2\,x)} \sin(3\,x)\, A\, x^2 - 12\,x\,\mathbf{e}^{(2\,x)} \sin(3\,x)\, B \\
&+ 18\,\mathbf{e}^{(2\,x)} \cos(3\,x)\, E\, x^2 + 12\,x\,\mathbf{e}^{(2\,x)} \cos(3\,x)\, F - 6\,\mathbf{e}^{(2\,x)} \sin(3\,x)\, C \\
&+ 6\,\mathbf{e}^{(2\,x)} \cos(3\,x)\, G
\end{aligned}
$$

```
> f:=lh - x^2*exp(2*x)*cos(3*x);
```

$$f := 2\,e^{(2x)}\cos(3x)\,B + 2\,e^{(2x)}\sin(3x)\,F + 6\,e^{(2x)}\cos(3x)\,A\,x + 6\,e^{(2x)}\sin(3x)\,E\,x - 18\,e^{(2x)}\sin(3x)\,A\,x^2 - 12\,x\,e^{(2x)}\sin(3x)\,B + 18\,e^{(2x)}\cos(3x)\,E\,x^2 + 12\,x\,e^{(2x)}\cos(3x)\,F - 6\,e^{(2x)}\sin(3x)\,C + 6\,e^{(2x)}\cos(3x)\,G - x^2\,e^{(2x)}\cos(3x)$$

```
> f1 := subs(sin(3*x)=s, cos(3*x)=c, exp(2*x)=e, f);
```

$$f1 := 2\,e\,c\,B + 2\,e\,s\,F + 6\,e\,c\,A\,x + 6\,e\,s\,E\,x - 18\,e\,s\,A\,x^2 - 12\,e\,s\,B\,x + 18\,e\,c\,E\,x^2 + 12\,e\,c\,F\,x - 6\,e\,s\,C + 6\,e\,c\,G - x^2\,e\,c$$

```
> f2 := simplify(f1/e);
```

$$f2 := 2\,c\,B + 2\,s\,F + 6\,c\,A\,x + 6\,s\,E\,x - 18\,s\,A\,x^2 - 12\,s\,B\,x + 18\,c\,E\,x^2 + 12\,c\,F\,x - 6\,s\,C + 6\,c\,G - x^2\,c$$

```
> f3 := collect(f2, [c,s]);
```

$$f3 := (6\,A\,x + 18\,E\,x^2 + 12\,F\,x + 2\,B + 6\,G - x^2)\,c + (-18\,A\,x^2 + 2\,F + 6\,E\,x - 6\,C - 12\,B\,x)\,s$$

```
> p1 := coeff(f3,c);
```

$$p1 := 6\,A\,x + 18\,E\,x^2 + 12\,F\,x + 2\,B + 6\,G - x^2$$

```
> p2 := coeff(f3,s);
```

$$p2 := -18\,A\,x^2 + 2\,F + 6\,E\,x - 6\,C - 12\,B\,x$$

```
> p1 := collect(p1,x);
```

$$p1 := (18\,E - 1)\,x^2 + (12\,F + 6\,A)\,x + 6\,G + 2\,B$$

```
> p2 := collect(p2, x);
```

$$p2 := -18\,A\,x^2 + (-12\,B + 6\,E)\,x + 2\,F - 6\,C$$

```
> eqns := { seq(coeff(p1,x,i)=0,i=0..2),
  seq(coeff(p2,x,i)=0,i=0..2)};
```

$$eqns := \{6\,G + 2\,B = 0,\ 12\,F + 6\,A = 0,\ 18\,E - 1 = 0,\ 2\,F - 6\,C = 0,\ -12\,B + 6\,E = 0,\ -18\,A = 0\}$$

```
> solns :=solve(eqns, {A,B,C,E,F,G});
```

$$solns := \left\{ C = 0, F = 0, A = 0, G = \frac{-1}{108}, E = \frac{1}{18}, B = \frac{1}{36} \right\}$$

```
> yp:=subs(solns,yt);
```

$$yp := \frac{1}{36} x^2 e^{(2x)} \cos(3x) + \frac{1}{18} e^{(2x)} \sin(3x) x^3 - \frac{1}{108} x e^{(2x)} \sin(3x)$$

```
> subs(y(x)=yp, deqn);
```

$$\frac{d^2}{dx^2}\left(\frac{1}{36} x^2 e^{(2x)} \cos(3x) + \frac{1}{18} e^{(2x)} \sin(3x) x^3 - \frac{1}{108} x e^{(2x)} \sin(3x) \right)$$

$$- 4 \left(\frac{d}{dx}\left(\frac{1}{36} x^2 e^{(2x)} \cos(3x) + \frac{1}{18} e^{(2x)} \sin(3x) x^3 - \frac{1}{108} x e^{(2x)} \sin(3x) \right)\right) + \frac{13}{36} x^2 e^{(2x)} \cos(3x) + \frac{13}{18} e^{(2x)} \sin(3x) x^3 - \frac{13}{108} x e^{(2x)} \sin(3x) =$$

$$x^2 e^{(2x)} \cos(3x)$$

```
> simplify(%);
```

$$x^2 e^{(2x)} \cos(3x) = x^2 e^{(2x)} \cos(3x)$$

```
>
```