
PROBLEM SET

Practice Problems for Exam #1

Math 2350, Fall 2004

Sept. 30, 2004

- You **must** show enough work to justify your answers. Unless otherwise instructed, give exact answers, not approximations (e.g., $\sqrt{2}$, not 1.414).
- This problem set has 8 problems.

Good luck!

Problem 1.

The position vector of a particle is given by $\mathbf{R}(t) = (t, t^2, t^3)$. Find the velocity and acceleration vectors of the particle. Find the speed of the particle at $t = 1$.

Problem 2.

The acceleration vector of a particle moving in space is $\mathbf{A}(t) = \mathbf{i} + 2t\mathbf{j} + \sin(t)\mathbf{k}$. The velocity vector of the particle at $t = 0$ is $\mathbf{V}_0 = \mathbf{j} + \mathbf{k}$ and the position vector at time $t = 0$ is $\mathbf{R}_0 = 2\mathbf{i}$. Find $\mathbf{R}(t)$, the position vector at an arbitrary time t .

Problem 3.

Alice stands at the edge of a cliff. From 100 feet above the ground she throws a baseball at 88 feet per second, at an angle of 30° above the horizontal. How far from the base of the cliff does the ball land? What is the time of flight? What is the ball's maximum height above the ground? (Neglect air resistance.) Give numerical answers accurate to two decimal places.

Problem 4.

A cannon has a muzzle speed of 550 feet per second. A shot is fired from ground level at an angle of 20° and overshoots the target by 50 feet. At what angle should the next shot be fired to hit the target (same muzzle speed)? Neglect air resistance. Give numerical answers accurate to two decimal places.

Problem 5.

Consider the curve parametrized in polar coordinates as $r = 5 \sin(\theta)$, $\theta = 2t$. Express the velocity and acceleration vectors in terms of \mathbf{u}_r and \mathbf{u}_θ . Find the speed of the curve.

Problem 6.

Consider the curve in the plane parametrized by $\mathbf{R}(t) = (t, t^2)$. Find the speed, the unit tangent vector, the curvature and the principal unit normal. Find the osculating circle at the point where $t = 1$. Find the scalar tangential and normal components of the acceleration as the moving particle goes through the point where $t = 1$.

Problem 7.

A stunt driver wants to drive a motorcycle around a vertical loop with a radius of 50 feet. How fast must the motorcycle be traveling at the top of the loop so it doesn't fall off?

Problem 8.

A satellite follows a stable circular orbit 100 miles above the surface of the earth. Find the orbital velocity and period of the satellite.

Use the following values: $g = 32 \text{ ft/sec}^2$, the radius of the earth is $R_e = 3963 \text{ mi}$, and $GM = gR_e^2$.
