EXAM

Exam #3
Take-home Exam

Math 2350, Fall 2004

Nov. 30, 2004

- This exam is due by 5:00 p.m. on Monday, December 6.
- Write all of your answers on separate sheets of paper. You can keep the exam questions.
- You **must** show enough work to justify your answers. Unless otherwise instructed, give exact answers, not approximations (e.g., $\sqrt{2}$, not 1.414).
- This exam has 9 problems. There are **400 points total**.
- I will e-mail you your grades. Please write your e-mail address on your exam paper, to confirm your address.

Good luck!

40 pts.

Problem 1. Us Lagrange Multipliers to find the absolute maximum and minimum of the function $f(x, y, z) = x^2 + yz$ on the spherical surface $x^2 + y^2 + z^2 = 4$.

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40 pts.

Problem 2. Let C be the curve formed by the intersection of the spherical surface $x^2+y^2+z^2 = 1$ and the plane $x+y+z = 1$. Use Lagrange multipliers to find the highest and lowest points on C, i.e., the points with the largest and smallest z-coordinates.

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40 pts.

Problem 3. Let R be the region in the xy-plane bounded by the curve $y = x^2$ and the line $y = 2x$. Set up iterated integrals for evaluating

$$\iint_R xy\, dA,$$

in both orders of integration. Evaluate one of these integrals.

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40 pts.

Problem 4. In each part, sketch the region of integration. Write down the equivalent iterated integral with the order of integration reversed and evaluate this integral.

A.

$$\int_0^4 \int_{-\sqrt{y}}^{\sqrt{y}} x\, dx\, dy.$$

B.

$$\int_0^1 \int_{2x}^2 y\, dy\, dx.$$

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40 pts.

Problem 5. Let R be the region in the first quadrant bounded by the coordinate axes and the circle of radius 1 (a quarter disk). Find the centroid of R using double integrals and polar coordinates.

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80 pts.

Problem 6. Let R be the region in three dimensional space bounded by the planes $z = 0$, $z = y$ and the cylinder $y = 1 - x^2$.

A. Set up an iterated integral for finding the volume of R, where the first integration is with respect to z.

B. Set up an iterated integral for finding the volume of R, where the first integration is with respect to y.

C. Set up an iterated integral for finding the volume of R, where the first integration is with respect to x.

D. Evaluate one of these integrals.

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40 pts.

Problem 7. Let R be the region bounded below by the xy-plane and above by the surface $z = 4 - x^2 - y^2$.

A. Find the z-coordinate of the centroid of R.

B. Find the moment of inertia of R for rotation about the z-axis (density=1).

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40 pts.

Problem 8. Let R be the region in three dimensional space that lies in the first octant and is bounded by the sphere $x^2 + y^2 + z^2 = a^2$ and the coordinate planes. Use spherical coordinates and triple integrals to find the centroid of R.

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40 pts.

Problem 9. Consider the region R in three dimensional space formed by taking the solid bounded above by the upper hemisphere of the sphere $x^2 + y^2 + z^2 = 4a^2$ and below by the xy-plane and then boring out a hole with a circular cross-section of radius a, centered on the z-axis.

Find the volume of R. Find the z-coordinate of the centroid of R.
