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# EXAM

Exam #3  
Take-home Exam

Math 1352, Fall 2004

November 30, 2004

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- This exam is due by 5:00 p.m. on Monday, December 6.
- Write all of your answers on separate sheets of paper. You can keep the exam questions.
- You **must** show enough work to justify your answers. Unless otherwise instructed, give exact answers, not approximations (e.g.,  $\sqrt{2}$ , not 1.414).
- This exam has 6 problems. There are **480 points total**.
- I will e-mail you your grades. Please write your e-mail address on your exam paper, to confirm your address.

Good luck!

100 pts.

**Problem 1.** In each part, find the integral. You must show how to apply one of the methods we learned in class.

A.

$$\int \frac{4x^2 - 2x + 3}{x^2(x^2 + 1)} dx.$$

B.

$$\int \frac{x^2}{(a^2 - x^2)^{3/2}} dx.$$

C.

$$\int x^3 \sqrt{a^2 - x^2} dx, \quad \text{Hint: simple substitution.}$$

D.

$$\int \frac{x^2}{\sqrt{4 + x^2}} dx.$$

E.

$$\int x^2 \sqrt{x^2 - 1} dx.$$

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40 pts.

**Problem 2.**

A rubber ball is dropped from a height of 12 feet. After each bounce, the ball goes up to  $3/4$  of the distance it previously fell. What is the total vertical distance traveled by the ball?

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40 pts.

**Problem 3.** In each part, determine if the series converges or diverges. If it converges, find the sum.

A.

$$\sum_{n=0}^{\infty} (-1)^n 5 \frac{2^n}{7^n}.$$

B.

$$\sum_{n=0}^{\infty} (-1)^n \frac{5^n}{4^n}.$$

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160 pts.

**Problem 4.** In each part, determine if the series is convergent or divergent. Be sure to say which test you are using and to show the details of how the test is applied.

A.

$$\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^6 + n^2 + 2}}.$$

B.

$$\sum_{n=1}^{\infty} \frac{n^3 + n^2 + 1}{2n^3 + 3n + 1}.$$

C.

$$\sum_{n=1}^{\infty} \frac{\sqrt{\ln(n)}}{n}.$$

D.

$$\sum_{n=1}^{\infty} \frac{n^3}{3^n}.$$

E.

$$\sum_{n=1}^{\infty} \frac{5^{2n}}{(2n)!}.$$

F.

$$\sum_{n=1}^{\infty} \frac{\sqrt{n!}}{7^n}.$$

G.

$$\sum_{n=1}^{\infty} \frac{n^2}{n + 3^n}.$$

H.

$$\sum_{n=1}^{\infty} \ln(1 + 1/n),$$

Hint: compare to  $\sum 1/n$ .

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60 pts.

**Problem 5.** In each part, determine if the series is absolutely convergent, conditionally convergent, or divergent. Be sure to say which tests you are using and to show the details of how the tests are applied.

A.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}.$$

B.

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n^4 + 1}.$$

C.

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(n+1)^2}{n^2}.$$

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80 pts.

**Problem 6.** In each part, you are given a power series. Find the interval of convergence of the power series and the radius of convergence. For each endpoint of the interval of convergence, determine the behavior of the series at that point.

A.

$$\sum_{n=1}^{\infty} \frac{(x+2)^n}{n}.$$

B.

$$\sum_{n=1}^{\infty} \frac{n!}{n^{2n}} x^n.$$

C.

$$\sum_{n=1}^{\infty} \frac{2^n}{n^2} x^n.$$

D.

$$\sum_{n=0}^{\infty} \frac{n!}{2^n} x^n.$$

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