PROBLEM SET

Practice Problems for Exam #2

Math 1352, Fall 2004

Nov. 5, 2004

ANSWERS

Problem 1.

A.

$$\int x\sqrt{2x+1}\,dx.$$

Answer:

Make the change of variables $u = 2x+1$, so $dx = (1/2)du$ and $x = (u-1)/2$. Then

$$\begin{aligned}\int x\sqrt{2x+1}\,dx &= \int \frac{u-1}{2}\sqrt{u}\frac{1}{2}\,du\\ &= \frac{1}{4}\int (u-1)u^{1/2}\,du\\ &= \frac{1}{4}\int [u^{3/2}-u^{1/2}]\,du\\ &= \frac{1}{4}\left[\frac{2}{5}u^{5/2}-\frac{2}{3}u^{3/2}\right]+C\\ &= \frac{1}{10}u^{5/2}-\frac{1}{6}u^{3/2}+C\\ &= \frac{1}{10}(2x+1)^{5/2}-\frac{1}{6}(2x+1)^{3/2}+C.\end{aligned}$$

B.

$$\int x\cos(2x)\,dx.$$

Answer:

Use the integration by parts formula

$$\boxed{\int uv'\,dx = uv - \int u'v\,dx} \tag{1.1}$$

with $u = x$ and $v' = \cos(2x)$. Then $u' = 1$ and

$$v = \int \cos 2x\,dx = \frac{1}{2}\sin(2x).$$

Applying the integration by parts formula we get

$$\begin{aligned}\int x\cos(2x)\,dx &= \frac{1}{2}x\sin(2x) - \int (1)\frac{1}{2}\sin(2x)\,dx\\ &= \frac{1}{2}x\sin(2x) - \frac{1}{2}\int \sin(2x)\,dx\\ &= \frac{1}{2}x\sin(2x) - \frac{1}{2}\left[-\frac{1}{2}\cos(2x)\right]+C\\ &= \frac{1}{2}x\sin(2x) + \frac{1}{4}\cos(2x)+C.\end{aligned}$$

C.

$$\int [\ln(x)]^2 \, dx.$$

Answer:

Use integration by parts with

$$u = [\ln(x)]^2, \qquad v' = 1$$

so

$$u' = \frac{2\ln(x)}{x}, \qquad v = x.$$

Then

$$\begin{aligned}\int [\ln(x)]^2 \, dx &= x[\ln(x)]^2 - \int \frac{2\ln(x)}{x} x \, dx \\ &= x[\ln(x)]^2 - 2\int \ln(x) \, dx.\end{aligned}$$

To do the integral $\int \ln(x) \, dx$ one would use integration by parts again. We did this problem in class, so I'll just use the result

$$\int \ln(x) \, dx = x\ln(x) - x + C.$$

Plugging this into the previous formula, we get

$$\int [\ln(x)]^2 \, dx = x[\ln(x)]^2 - 2x\ln(x) - 2x + C.$$

D.

$$\int x^6 \ln(x) \, dx$$

Answer:

Use integration by parts with $u = \ln(x)$ and $v' = x^6$, so $u' = 1/x$ and $v = x^7/7$. Then

$$\begin{aligned}\int x^6 \ln(x) \, dx &= \frac{1}{7}x^7 \ln(x) - \int \frac{1}{x}\frac{1}{7}x^7 \, dx \\ &= \frac{1}{7}x^7 \ln(x) - \frac{1}{7}\int x^6 \, dx \\ &= \frac{1}{7}x^7 \ln(x) - \frac{1}{49}x^7 + C.\end{aligned}$$

E.

$$\int \sin^2(x)\cos^5(x)\,dx.$$

Answer:

Since the power of cosine is odd, borrow a cosine and write the integral as

$$\int \sin^2(x)\cos^4(x)\cos(x)\,d(x).$$

In this integral make the change of variables, $u = \sin(x)$, so $du = \cos(x)\,dx$. Thus,

$$\begin{aligned}\int \sin^2(x)\cos^4(x)\cos(x)\,d(x) &= \int (1-\cos^2(x))\cos^4(x)\cos(x)\,dx\\ &= \int (1-u^2)u^4\,du\\ &= \int [u^4-u^6]\,du\\ &= \frac{1}{5}u^5 - \frac{1}{7}u^7 + C\\ &= \frac{1}{5}\sin^5(x) - \frac{1}{7}\sin^7(x) + C.\end{aligned}$$

F.

$$\int \tan^3(x)\sec^3(x)\,dx.$$

Answer:

Borrow a secant and a tangent, and write the integral as

$$\int \tan^3(x)\sec^3(x)\,dx = \int \tan^2(x)\sec^2(x)\sec(x)\tan(x)\,dx = \int (\sec^2(x){-}1)\sec^2(x)\sec(x)\tan(x)\,dx.$$

Make the change of variables $u = \sec(x)$ so $du = \sec(x)\tan(x)\,d(x)$. Thus,

$$\begin{aligned}\int (\sec^2(x)-1)\sec^2(x)\sec(x)\tan(x)\,dx &= \int (u^2-1)u^2\,du\\ &= \int (u^4-u^2)\,du\\ &= \frac{1}{5}u^5 - \frac{1}{3}u^3 + C\\ &= \frac{1}{5}\sec^5(x) - \frac{1}{3}\sec^3(x) + C.\end{aligned}$$

G.

$$\int \tan^2(x)\sec^3(x)\,dx.$$

Answer:

Write the integral as

$$\begin{aligned}\int \tan^2(x)\sec^3(x)\,dx &= \int (\sec^2 - 1)\sec^3(x)\,dx \\ &= \int [\sec^5 x - \sec^3(x)]\,dx \end{aligned}\tag{1.2}$$

Now use formula (5) from the reduction formula's sheet, which reads

$$\int \sec^n(x)\,dx = \frac{1}{n-1}\sec^{n-2}(x)\tan(x) + \frac{n-2}{n-1}\int \sec^{n-2}(x)\,dx.\tag{1.3}$$

Applying this formula with $n = 5$ gives

$$\int \sec^5(x)\,dx = \frac{1}{4}\sec^3(x)\tan(x) + \frac{3}{4}\int \sec^3(x)\,dx.\tag{1.4}$$

Plugging this into (1.2) gives

$$\begin{aligned}\int \tan^2(x)\sec^3(x)\,dx &= \int \sec^5(x)\,dx - \int \sec^3(x)\,dx \\ &= \frac{1}{4}\sec^3(x)\tan(x) - \frac{1}{4}\int \sec^3(x)\,dx \end{aligned}\tag{1.5}$$

Applying (1.3) with $n = 3$ gives

$$\begin{aligned}\int \sec^3(x)\,dx &= \frac{1}{2}\sec x\tan(x) + \frac{1}{2}\int \sec(x)\,dx \\ &= \frac{1}{2}\sec()\tan(x) + \frac{1}{2}\ln|\sec(x) + \tan(x)| + C \end{aligned}\tag{1.6}$$

Plugging this into (1.5), we get

$$\begin{aligned}\int \tan^2(x)\sec^3(x)\,dx &= \frac{1}{4}\sec^3(x)\tan(x) - \frac{1}{4}\int \sec^3(x)\,dx \\ &= \frac{1}{4}\sec^3(x)\tan(x) - \frac{1}{4}\left[\frac{1}{2}\sec(x)\tan(x) + \frac{1}{2}\ln|\sec(x) + \tan(x)|\right] + C \\ &= \frac{1}{4}\sec^3(x)\tan(x) - \frac{1}{8}\sec(x)\tan(x) - \frac{1}{8}\ln|\sec(x) + \tan(x)| + C. \end{aligned}$$

H.

$$\int \frac{1}{2x^2+4x+10}\,dx$$

Answer:

Complete the square in the denominator. So, we write

$$2x^2+4x+10 = 2(x^2+2x)+10.$$

Taking half the coefficient of x in x^2+2x we get 1, so we add and subtract 1^2 inside the parenthesis to get

$$\begin{aligned} 2x^2+4x+10 &= 2(x^2+2x+1-1)+10 \\ &= 2[(x+1)^2-1]+10 \\ &= 2(x+1)^2-2+10 \\ &= 2(x+1)^2+8. \end{aligned}$$

Thus, we have

$$\int \frac{1}{2x^2+4x+10}\,dx = \int \frac{dx}{2(x+1)^2+8}.$$

In this integral make the change of variables $u = x+1$, so $du = dx$. Then we have

$$\begin{aligned} \int \frac{1}{2x^2+4x+10}\,dx &= \int \frac{dx}{2(x+1)^2+8} \\ &= \int \frac{du}{2u^2+8} \\ &= \frac{1}{8}\int \frac{du}{u^2/4+1} \\ &= \frac{1}{8}\int \frac{du}{(u/2)^2+1}, & w = u/2,\ du = 2\,dw \\ &= \frac{1}{8}\int \frac{2\,dw}{w^2+1} \\ &= \frac{1}{4}\int \frac{dw}{w^2+1} \\ &= \frac{1}{4}\tan^{-1}(w)+C \\ &= \frac{1}{4}\tan^{-1}(u/2)+C \\ &= \frac{1}{4}\tan^{-1}\left(\frac{x+1}{2}\right)+C. \end{aligned}$$

I.

$$\int \frac{1}{x\sqrt{a^2-x^2}}\,dx.$$

Answer:

Make the trig substitution $x = a\sin(\theta)$, so $dx = \cos(\theta)\,d\theta$ and

$$a^2 - x^2 = a^2 - a^2\sin^2(\theta) = a^2(1-\sin^2(\theta)) = a^2\cos^2(\theta)$$

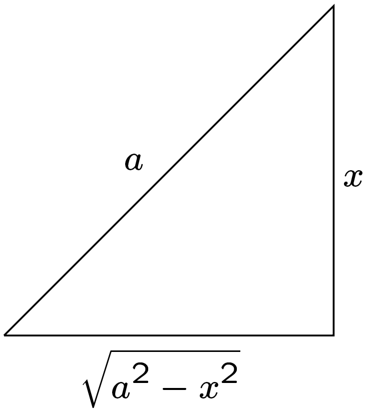
so

$$\sqrt{a^2-x^2} = a\cos(\theta).$$

Then we have

$$\begin{aligned}\int \frac{1}{x\sqrt{a^2-x^2}}\,dx &= \int \frac{1}{a\sin(\theta)a\cos(\theta)}a\cos(\theta)\,d\theta\\ &= \frac{1}{a}\int \frac{1}{\sin(\theta)}\,d\theta\\ &= \frac{1}{a}\int \csc(\theta)\,d\theta\\ &= -\ln|\csc(x)+\cot(x)| + C.\end{aligned}$$

We have $\sin(\theta) = x/a$, so the reference triangle looks like this



Thus we have

$$\csc(\theta) = \frac{1}{\sin(\theta)} = \frac{a}{x},$$
$$\cot(\theta) = \frac{\sqrt{a^2-x^2}}{x}.$$

Plugging this in gives

$$\int \frac{1}{x\sqrt{a^2-x^2}}\,dx = -\ln\left|\frac{a}{x} + \frac{\sqrt{a^2-x^2}}{x}\right| + C.$$

J.

$$\int \frac{dx}{(a^2+x^2)^{3/2}}.$$

Answer:

Make the trig substitution $x = a\tan(\theta)$, so $dx = \sec^2(\theta)\,d\theta$ and

$$a^2 + x^2 = a^2 + a^2\tan^2(\theta) = a^2(1+\tan^2(\theta)) = a^2\sec^2(\theta).$$

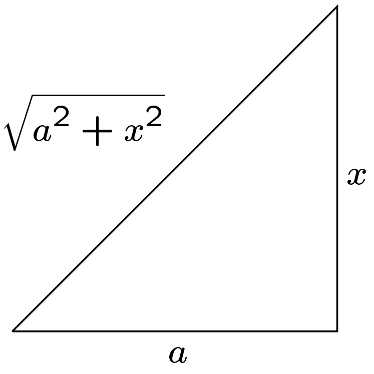
Thus, we have

$$(a^2+x^2)^{3/2} = (a^2\sec^2(\theta))^{3/2} = a^3\sec^3(\theta)$$

We calculate that

$$\begin{aligned}\int \frac{dx}{(a^2+x^2)^{3/2}}. &= \int \frac{a\sec^2(\theta)\,d\theta}{a^3\sec^3(\theta)} \\ &= \frac{1}{a^2}\int \frac{d\theta}{\sec(\theta)} \\ &= \frac{1}{a^2}\int \cos(\theta)\,d\theta \\ &= \frac{1}{a^2}\sin(\theta) + C.\end{aligned}$$

We have $\tan(\theta) = x/a$, so the reference triangle looks like this



Reading off $\sin(\theta)$ from the triangle we get

$$\int \frac{dx}{(a^2+x^2)^{3/2}} = \frac{1}{a^2}\frac{x}{\sqrt{a^2+x^2}} + C.$$

K.

$$\int \frac{\sqrt{x^2-a^2}}{x}\,dx.$$

Answer:

For the case $x > 0$, make the trig substitution $x = a\sec(\theta)$, so $dx = a\sec(\theta)\tan(\theta)\,d\theta$ and

$$x^2 - a^2 = a^2\sec^2(\theta) - a^2 = a^2(\sec^2(\theta)-1) = a^2\tan^2(\theta),$$

so

$$\sqrt{x^2-a^2} = a\tan(\theta).$$

Then we have

$$\begin{aligned}
\int \frac{\sqrt{x^2-a^2}}{x}\,dx &= \int \frac{a\tan(\theta)}{a\sec(\theta)} a\sec(\theta)\tan(\theta)\,d\theta \\
&= a\int \tan^2(\theta)\,d\theta \\
&= a\int [\sec^2(\theta) - 1]\,d\theta \\
&= a\tan(\theta) - a\theta + C \\
&= a\sqrt{x^2-a^2} - a\sec^{-1}(x/a) + C.
\end{aligned}$$

For the case $x < 0$, make the substitution $x = -u$, so $u > 0$, and $dx = -du$. Then

$$\begin{aligned}
\int \frac{\sqrt{x^2-a^2}}{x}\,dx &= \int \frac{\sqrt{(-u)^2-a^2}}{-u}(-1)\,du \\
&= \int \frac{\sqrt{u^2-a^2}}{u}\,du \\
&= a\sqrt{u^2-a^2} - a\sec^{-1}(u/a) + C, && \text{(from above)} \\
&= a\sqrt{(-x)^2-a^2} - a\sec^{-1}(-x/a) + C. \\
&= a\sqrt{x^2-a^2} - a\sec^{-1}(-x/a) + C.
\end{aligned}$$

Thus, we have

$$\int \frac{\sqrt{x^2-a^2}}{x}\,dx = \begin{cases} a\sqrt{x^2-a^2} - a\sec^{-1}(x/a) + C, & x > 0 \\ a\sqrt{x^2-a^2} - a\sec^{-1}(-x/a) + C, & x < 0. \end{cases}$$

We can put these into one formula as

$$\int \frac{\sqrt{x^2-a^2}}{x}\,dx = a\sqrt{x^2-a^2} - a\sec^{-1}\left(\frac{|x|}{a}\right) + C.$$

L.

$$\int \frac{2x^2+x+1}{x(x+1)^2)}\,dx.$$

Answer:

Use Partial Fractions. The form of the partial fraction decomposition is

$$\frac{2x^2+x+1}{x(x+1)^2)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}.$$

Clearing the denominators gives the equation

$$2x^2+x+1 = A(x+1)^2 + Bx(x+1) + Cx. \tag{1.7}$$

Expanding the right-hand side gives

$$\begin{aligned} 2x^2+x+1 &= A(x+1)^2 + Bx(x+1) + Cx \\ &= Ax^2 + 2Ax + A + Bx^2 + Bx + C \\ &= (A+B)x^2 + (2A+B+C)x + A. \end{aligned}$$

Equating coefficents of powers of x gives the system of equations

$$\begin{aligned} A+B &= 2 \\ 2A+B+C &= 1 \\ A &= 1. \end{aligned}$$

Setting $x=0$ in (1.7) gives $A=1$ (which we knew) and setting $x=-1$ in (1.7) gives $C=-2$. The first equation then shows $B=1$. Thus, we have

$$\frac{2x^2+x+1}{x(x+1)^2)} = \frac{1}{x} + \frac{1}{x+1} - \frac{2}{(x+1))^2}.$$

Then we have

$$\begin{aligned} \int \frac{2x^2+x+1}{x(x+1)^2)}\,dx &= \int \left[\frac{1}{x} + \frac{1}{x+1} - \frac{2}{(x+1))^2}\right] dx \\ &= \ln|x| + \ln|x+1| + \frac{2}{x+1} + C. \end{aligned}$$

M.

$$\int \frac{2x^2+2x+1}{x(x^2+1)}\,dx$$

Answer:

The form of the partial fraction decomposition is

$$\frac{2x^2+2x+1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}.$$

Clearing denominators

$$\begin{aligned} 2x^2 + 2x + 1 &= A(x^2+1) + (Bx + C)x \\ &= Ax^2 + A + Bx^2 + Cx \\ &= (A+B)x^2 + Cx + A. \end{aligned}$$

Equating coefficients gives the system of equations

$$\begin{aligned} A + B &= 2 \\ C &= 2 \\ A &= 1, \end{aligned}$$

so, clearly, $B = 1$.

Then we have

$$\begin{aligned} \int \frac{2x^2+2x+1}{x(x^2+1)}\,dx &= \int \left[\frac{1}{x} + \frac{x+2}{x^2+1}\right] dx \\ &= \int \frac{dx}{x} + \int \frac{x}{x^2+1}\,dx + 2\int \frac{dx}{x^2+1} \\ &= \ln|x| + \frac{1}{2}\ln(x^2+1) + 2\tan^{-1}(x) + C, \end{aligned}$$

where the middle integral can be done with a simple substitution (or "guess and correct").

Problem 2. In each part, give the *form* of the partial fraction decomposition. This is a formula involving undetermined coefficients. **Do not find the coefficients!** (No calculation is required).

A.

$$\frac{x^3 + 2x + 1}{(x-1)(x-2)(x+3)}$$

Answer:

$$\frac{x^3 + 2x + 1}{(x-1)(x-2)(x+3)} = \boxed{\frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x+3}}\,.$$

B.

$$\frac{1}{x(x^2+1)}$$

Answer:

$$\frac{1}{x(x^2+1)} = \boxed{\frac{A}{x} + \frac{Bx+C}{x^2+1}}\,.$$

C.

$$\frac{x^4+1}{x(x^2+1)^2}$$

Answer:

$$\frac{x^4+1}{x(x^2+1)^2} = \boxed{\frac{A}{x} + \frac{Bx+C}{x^2+1} + \frac{Dx+E}{(x^2+1)^2}}\,.$$

D.

$$\frac{x^3}{(x-2)^2(x+2)^2(x-1)}$$

Answer:

$$\begin{aligned}&\frac{x^3}{(x-2)^2(x+2)^2(x-1)}\\ &\qquad = \boxed{\frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{x+2} + \frac{D}{(x+2)^2} + \frac{E}{x-1}}\,.\end{aligned}$$

Problem 3. In each part, solve the differential equation.

A.

$$\frac{dy}{dx} - \frac{2}{x}y = x^2.$$

Answer:

This is a first order linear equation. Comparing with the standard form

$$\frac{dy}{dx} + P(x)y = Q(x)$$

we see that $P(x) = -2/x$. We have

$$\int P(x)\,dx = -2\ln(x) = \ln(1/x^2).$$

Thus, the integrating factor is

$$I(x) = e^{\int P(x)\,dx} = e^{\ln(1/x^2)} = \frac{1}{x^2}.$$

Multiplying the differential equation through by the integrating factor gives

$$\frac{1}{x^2}\frac{dy}{dx} - \frac{2}{x^3}y = 1.$$

This is the same as

$$\frac{d}{dx}\left(\frac{1}{x^2}y\right) = 1.$$

(that's what the integrating factor does,and you can check it with the product rule). Integrating both sides gives

$$\frac{1}{x^2}y = x + C,$$

so the solution is

$$y = x^3 + Cx^2.$$

B.

$$\frac{dy}{dx} + 4y = 2e^x.$$

Answer:

The integrating factor is e^{4x}. The solution is $y = (2/5)e^x + Ce^{-4x}$.

C.

$$\frac{dy}{dx} = y(y-1).$$

Answer:

The variables separate as

$$\frac{dy}{y(y-1)} = dx$$

Integrating both sides we get

$$\int \frac{dy}{y(y-1)} = \int dx = x + C$$

To do the integral on the left, we use partial fractions. The form of the partial fraction decomposition is

$$\frac{1}{y(y-1)} = \frac{A}{y-1} + \frac{B}{y}.$$

Clearing the denominators gives

$$1 = Ay + B(y-1)$$

Setting $y = 0$ shows $B = -1$ and setting $y = 1$ shows $A = 1$. Thus, we have

$$\begin{aligned}\int \frac{dy}{y(y-1)} &= \int \left[\frac{1}{y-1} - \frac{1}{y}\right] dx \\ &= \ln|y-1| - \ln|y| \qquad\qquad \text{(don't need } C \text{ here)} \\ &= \ln\left|\frac{y-1}{y}\right|.\end{aligned}$$

Thus, we have

$$\ln\left|\frac{y-1}{y}\right| = x + C$$

Taking the exponential of both sides gives

$$\left|\frac{y-1}{y}\right| = e^{x+C} = e^C e^x$$

and so

$$\frac{y-1}{y} = \pm e^C e^x.$$

Since C is arbitrary, e^C is an arbitrary positive number, so $\pm e^C$ is an arbitrary constant, call it C again, So we have

$$\frac{y-1}{y} = Ce^x$$

Solving this for y gives

$$y = \frac{1}{1 - Ce^x}.$$

Since $-C$ is an arbitrary constant, we could write this as

$$y = \frac{1}{1 + Ce^x}$$

for an arbitrary constant C.

Problem 4. A tank contains 200 gallons of water. At the beginning of the problem there are 100 pounds of salt dissolved in the tank. Salt water enters the tank at a rate of 5 gallons per minute. Each gallon of incoming salt water contains 1 pound dissolved salt. The mixture in the tank is kept prosaically uniform by stirring, and 5 gallons of salt water is drained from the tank per minute. Find the amount (number of pounds) of salt in the tank as a function of time. How long will it be until there are 175 pounds of salt in the tank?

Answer:
Let $y = y(t)$ be the number of pounds of salt in the tank. Thus, $y(0) = 100$. There are 5 pounds of salt entering the tank per minute.

The concentration of salt in the tank is $y/200$. Five gallons of this mixture leaves the tank per minute so, salt is leaving the thank at a rate of $5(y/200) = y/40$ pounds per minute. Thus, we have

$$\frac{dy}{dt} = 5 - \frac{1}{40}y.$$

This differential equation can be solved by separation of variables or as a first order linear equation. If we take the latter approach, we rewrite the equation as

$$\frac{dy}{dt} + \frac{1}{40} y = 5.$$

The integrating factor is $e^{t/40}$ and the solution is

$$y = 200 + Ce^{-t/40}.$$

Setting $t = 0$ in this equation and using $y(0) = 100$ we have

$$100 = 200 + C,$$

so $C = -100$. Thus, the amount of salt in the tank is given as a function of time by

$$y = 200 - 100e^{-t/40}.$$

To find the time at which there are 175 pounds of salt in the tank, set $y = 175$ in the last equation, which gives

$$175 = 200 - 100e^{-t/40},$$

which simplifies to

$$1/4 = e^{-t/40}.$$

Taking log of both sides gives

$$\ln(1/4) = -t/40,$$

and so

$$t = -40 \ln(1/4) \approx 55.45$$

minutes.

Problem 5. In each part, determine if the improper integral converges or diverges. If it converges, find the value.

A.

$$\int_0^\infty e^{-2x}\, dx$$

Answer:

First, we figure out the indefinite integral

$$\int e^{-2x}\, dx = -\frac{1}{2} e^{-2x} + C. \tag{5.1}$$

The integral in the problem is improper because the upper limit is infinity so, by definition,

$$\begin{aligned}\int_0^\infty e^{-2x}\,dx &= \lim_{b\to\infty}\int_0^b e^{-2x}\,dx\\ &= \lim_{b\to\infty}\left[-\frac{1}{2}e^{-2x}\right]_0^b, && \text{(by (5.1))}\\ &= \lim_{b\to\infty}\left[-\frac{1}{2}e^{-2b}+\frac{1}{2}e^{2(0)}\right]\\ &= \lim_{b\to\infty}\left[\frac{1}{2}-\frac{1}{2}e^{-2b}\right]\\ &= \boxed{\frac{1}{2}},\end{aligned}$$

since $\lim_{b\to\infty} e^{-2b} = 0$. Thus, this integral $\boxed{\text{converges}}$.

B.

$$\int_0^1 \frac{1}{x^2}\,dx$$

Answer:

Note the indefinite integral

$$\int \frac{1}{x^2}\,dx = -\frac{1}{x}+C. \tag{5.2}$$

The integral in the problem is improper because the integrand is undefined at 0, the lower endpoint of the interval of integration. Thus, by definition,

$$\begin{aligned}\int_0^1 \frac{1}{x^2}\,dx &= \lim_{a\to 0^+}\int_a^1 \frac{1}{x^2}\,dx\\ &= \lim_{a\to 0^+}\left[-\frac{1}{x}\right]_a^1 && \text{(by (5.2))}\\ &= \lim_{a\to 0^+}\left[-1+\frac{1}{a}\right]\\ &= \infty,\end{aligned}$$

since $\lim_{a\to 0^+} 1/a = \infty$. Thus, this integral $\boxed{\text{diverges}}$.
