MATH 1352: Review Final Exam

Time = 150 minutes; Total points = 100;

- This review is intended to highlight the topics being examined, to make sure that all sections have the same information regarding the final.
- The included sections are: 6.1 – 6.5; 7.1 – 7.7; 8.1 – 8.8; and 9.1 – 9.4.
- Specific topic omitted: *Surface Area* (in both cartesian and polar coordinates) in Section 6.4.
- Only improper integrals of the type $\int_a^\infty f(x)\,dx$, will be required to be solved, though the students should be aware of other types of improper integrals.
- If an instructor has not covered the material for a problem, he might assign an alternate problem during the exam.
- Calculators are to be used at the discretion of the instructor.

1. Solve the following problems:

   i. Identify the improper integral(s) among if any (give a short reason):

      (i) $\int_0^5 \frac{x-2}{(x+2)^2}\,dx$

      (ii) $\int \frac{1}{(x-2)^2}\,dx$

      (iii) $\int_0^{2\pi} \tan x\,dx$.

   ii. Is the following sum correct - give a short mathematical reason: $1+\pi+\pi^2+\cdots = \frac{1}{1-\pi}$.

   iii. Do you agree with the following computation - give a short mathematical reason: $\int_{-1}^1 \frac{1}{x^2}\,dx = \int_{-1}^1 x^{-2}\,dx = \left.\frac{x^{-1}}{-1}\right|_{-1}^1 = -2.$

2. Solve **one** of the following two problems:

a. Find the x co-ordinate of the centroid of the region in the first quadrant bounded by the curves $y = x^2$ and $y = \sqrt{x}$.

b. (i) Find the point of intersection of the curves $r = 4\,\sin\theta$ and $r = 2$.

(ii) Find the area enclosed by one loop of $r = 2\,\sin 2\,\theta$.

3. Solve **one** of the following two problems:

a. According to Coulomb's law of physics, two similarly charged particles repel each other with a force inversely proportional to the square of the distance between them. Suppose the force is 12 dynes when they are 5 cm apart. How much work is done in moving one particle from a distance of 10 cm to a distance of 8 cm from the other?

b. Suppose that a ball is dropped from a height h and it rises 7 % of the distance it had previously fallen. If it travels a total distance of 21 ft, what is h?

4. Solve **one** of the following two problems:

a. Find the solution of the differential equation $x^4 \frac{dy}{dx} + 3\,x^3\,y = 5$ that passes through $(e, 1)$. What is the value of y for $x = -1$?

b. Find the sum of the series: $\sum_{k=1}^{\infty} \frac{2k+1}{k^2\,(k+1)^2}$.

5. Find the following integrals:

i. $\int \frac{dx}{\sqrt{x^2+8x+3}}$.

ii. $\int \frac{x^2+4x+4}{x^2+x+1}\,dx$.

iii. $\int \sin\sqrt{x}\,dx$

6. Find whether the sequence $\{\left(\frac{\ln n}{n}\right)^{\frac{1}{n}}\}$ converges or diverges.

7. Does the series $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{k}{k^2+1}$ converge absolutely, conditionally or diverges?

8. Test the following series for convergence. State the name of the tests that you have used to arrive at your conclusion:

   i. $\sum_{k=2}^{\infty} \left(\frac{k^k}{\ln^k k}\right)$.

   ii. $\sum_{k=1}^{\infty} \frac{\sqrt{k+1}}{k^{(k+0.5)}}$

   iii. $\sum_{k=1}^{\infty} \frac{(k+1)^3}{k^{\frac{9}{2}}}$.

   iv. $\sum_{k=2}^{\infty} \frac{1}{k \ln k}$.

9. Find the interval of convergence for the power series $\sum_{k=1}^{\infty} \frac{(x+2)^{2k}}{3^k}$.

10. Find the first four terms of the Taylor series of the function $\frac{1}{2-x}$ at $c = 5$.

11. Given $u = \mathbf{j} - 3\mathbf{k}$, and $v = -\mathbf{i} + \mathbf{j} + \mathbf{k}$.

   i. Find $u \cdot v$.

   ii. Find the angle between u and v.

   iii. Find the area of the parallelogram determined by u and v.

   iv. Find a unit vector normal to u and v.

   v. If $w = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ find the scalar triple product of u,v and w.