

40 pts.

Problem 1. Generate a large random matrix A and a large random vector in matlab with the commands $A = \text{round}(10*\text{rand}(2000))$; and $b = \text{round}(10*\text{rand}(2000,1))$; (note the semi-colon, you don't want to print these!) Two methods of solving the system $Ax = b$ are

```
x1 = A\b;  
x2= inv(A)*b;
```

- A. Which method is faster? (Use the commands `tic` and `toc`.)
 - B. Which method gives a smaller residual $r = b - Ax$, when the size of the residual is measured with the function `norm`?
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80 pts.

Problem 2.

- A. What does `A\b` do when the system $Ax = b$ is inconsistent? (Look it up in the matlab help.)
- B. Suppose that you are given vectors of numbers

$$x = [x_1 \ x_2 \ \dots \ x_n]$$
$$y = [y_1 \ y_2 \ \dots \ y_n].$$

How would you express the system of equations

$$y_i = ax_i + b, \quad i = 1, \dots, n$$

in matrix form (the unknowns are a and b)?

- C. Given the data

$$x = [0 \ 0.2 \ 0.4 \ 0.6 \ 0.8 \ 1.0]$$
$$y = [2.24 \ 2.59 \ 3.27 \ 3.83 \ 4.36 \ 5.15],$$

express the system

$$y_i = ax_i + b, \quad i = 1, \dots, 6$$

in matrix form and solve the system for a and b using the backslash operator in matlab.

- D. Using the values of a and b from the last part of the problem graph the line $y = ax + b$ and the data points (x_i, y_i) ($i = 1, \dots, 6$) on the same graph.
- E. For general data vectors x and y of length n , the correlation coefficient r measures how well the line fits the data. The formula for r is

$$r = \frac{n \sum_{i=1}^n x_i y_i - S_x S_y}{\sqrt{n \sum_{i=1}^n x_i^2 - S_x^2} \sqrt{n \sum_{i=1}^n y_i^2 - S_y^2}}$$

where

$$S_x = \sum_{i=1}^n x_i, \quad S_y = \sum_{i=1}^n y_i.$$

Calculate the regression coefficient for the data given in part C of the problem, without using any loops. (Hint: see the function `sum`.)

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Problem 3. In matlab, construct the vector

$$v = [0 \ 2 \ 0 \ 4 \ 0 \ 6 \ 0 \ \dots \ 18 \ 0 \ 20]$$

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Problem 4. Write matlab code to take a vector x and construct a vector y which contains the nonzero elements of x . Give an example.

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Problem 5. Given a vector a of length n , write matlab code to generate the matrix

$$\begin{bmatrix} 1 & 1 & \dots & 1 \\ a_1 & a_2 & \dots & a_n \\ a_1^2 & a_2^2 & \dots & a_n^2 \\ \vdots & \vdots & \ddots & \vdots \\ a_1^{n-1} & a_2^{n-1} & \dots & a_n^{n-1} \end{bmatrix}.$$

Give an example.

100 pts.

Problem 6. Generate a random matrix in matlab with the code $A = \text{round}(10*\text{rand}(6))$. In each part, use matlab code to produce the required matrix. **Do not use any loops.**

- A. The matrix B constructed by deleting row 5 and column 2 from A .
 - B. The matrix C consisting of the even numbered rows of A .
 - C. The matrix D obtained by replacing row 3 of A with seven times row 3 of A .
 - D. The matrix E obtained by replacing row 2 of A by row 2 of A plus 3 times row 5 of A .
 - E. The matrix F obtained by setting the entries in columns 1, 3 and 4 of A to zero.
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EXAM

Exam III

Math 5365-01, Summer II, 2003

August 6, 2002

- Write all of your answers on separate sheets of paper. You can keep the exam questions when you leave. You may leave when finished.
- You **must** show enough work to justify your answers.
- This exam has 6 problems. There are **340 points total**.

Good luck!