Solutions to Practice Exam #2, Computer part

Problem 1

```
> with(linalg):
```

```
Warning, the protected names norm and trace have been redefined and
unprotected
```

```
> A := matrix([
> [-3, -8, 10],
> [-4, -8, 11],
> [-4, -10, 13]]);
```

$$A := \begin{bmatrix} -3 & -8 & 10 \\ -4 & -8 & 11 \\ -4 & -10 & 13 \end{bmatrix}$$

First, find the eigenvalues

```
> eigenvals(A);
```

$$-1,\ 1,\ 2$$

Find the eigenvectors

```
> eigenvects(A);
```

$$[-1,\ 1,\ \{[1,\ 1,\ 1]\}],\ [2,\ 1,\ \{\left[\frac{4}{5},\ 1,\ \frac{6}{5}\right]\}],\ [1,\ 1,\ \{\left[\frac{1}{2},\ 1,\ 1\right]\}]$$

```
> v1 := vector([1/2, 1, 1]);
```

$$v1 := \left[\frac{1}{2},\ 1,\ 1\right]$$

```
> v2 := vector([4/5, 1, 6/5]);
```

$$v2 := \left[\frac{4}{5},\ 1,\ \frac{6}{5}\right]$$

```
> v3 := vector([1, 1, 1]);
```

$$v3 := [1,\ 1,\ 1]$$

Find the diagonalizing matrix P

```
> P := augment(v1,v2,v3);
```

$$P := \begin{bmatrix} \frac{1}{2} & \frac{4}{5} & 1 \\ 1 & 1 & 1 \\ 1 & \frac{6}{5} & 1 \end{bmatrix}$$

```
> Dm := evalm(inverse(P)&*A&*P);
```

$$Dm := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Find the expoential of Dm

```
> De := diag(exp(t), exp(2*t), exp(-t));
```

$$De := \begin{bmatrix} e^{t} & 0 & 0 \\ 0 & e^{(2\,t)} & 0 \\ 0 & 0 & e^{(-t)} \end{bmatrix}$$

Find the exponential of A

```
> expA := evalm(P&*De&*inverse(P));
```

$$expA := \begin{bmatrix} -e^{t} + 2\,e^{(-t)} & 2\,e^{t} - 4\,e^{(2\,t)} + 2\,e^{(-t)} & -e^{t} + 4\,e^{(2\,t)} - 3\,e^{(-t)} \\ -2\,e^{t} + 2\,e^{(-t)} & 4\,e^{t} - 5\,e^{(2\,t)} + 2\,e^{(-t)} & -2\,e^{t} + 5\,e^{(2\,t)} - 3\,e^{(-t)} \\ -2\,e^{t} + 2\,e^{(-t)} & 4\,e^{t} - 6\,e^{(2\,t)} + 2\,e^{(-t)} & -2\,e^{t} + 6\,e^{(2\,t)} - 3\,e^{(-t)} \end{bmatrix}$$

check that our computation is correct

```
> expAprime :=map(diff, expA, t);
```

$$expAprime := \begin{bmatrix} -e^{t} - 2\,e^{(-t)} & 2\,e^{t} - 8\,e^{(2\,t)} - 2\,e^{(-t)} & -e^{t} + 8\,e^{(2\,t)} + 3\,e^{(-t)} \\ -2\,e^{t} - 2\,e^{(-t)} & 4\,e^{t} - 10\,e^{(2\,t)} - 2\,e^{(-t)} & -2\,e^{t} + 10\,e^{(2\,t)} + 3\,e^{(-t)} \\ -2\,e^{t} - 2\,e^{(-t)} & 4\,e^{t} - 12\,e^{(2\,t)} - 2\,e^{(-t)} & -2\,e^{t} + 12\,e^{(2\,t)} + 3\,e^{(-t)} \end{bmatrix}$$

```
> aexpa := evalm(A&*expA);
```

$$aexpa := \begin{bmatrix} -e^{t} - 2\,e^{(-t)} & 2\,e^{t} - 8\,e^{(2\,t)} - 2\,e^{(-t)} & -e^{t} + 8\,e^{(2\,t)} + 3\,e^{(-t)} \\ -2\,e^{t} - 2\,e^{(-t)} & 4\,e^{t} - 10\,e^{(2\,t)} - 2\,e^{(-t)} & -2\,e^{t} + 10\,e^{(2\,t)} + 3\,e^{(-t)} \\ -2\,e^{t} - 2\,e^{(-t)} & 4\,e^{t} - 12\,e^{(2\,t)} - 2\,e^{(-t)} & -2\,e^{t} + 12\,e^{(2\,t)} + 3\,e^{(-t)} \end{bmatrix}$$

```
> evalm(expAprime - aexpa);
```

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

```
> simplify(map2(subs, t=0, expA));
```

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Problem 2

```
> N := matrix([
> [0, 1, 0, 0],
> [0, 0, 1, 0],
> [0, 0, 0, 1],
> [0, 0, 0, 0]]);
```

$$N := \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

```
> evalm(N^2); evalm(N^3); evalm(N^4);
```

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$N^4 = 0$

, so N is nilpotent.

Find the exponential of N:

```
> id4 := diag(1,1,1,1);
```

$$id4 := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

```
> expN := evalm(id4 + t*N + (t^2/2!)*N^2 + (t^3/3!)*N^3);
```

$$expN := \begin{bmatrix} 1 & t & \frac{1}{2}t^2 & \frac{1}{6}t^3 \\ 0 & 1 & t & \frac{1}{2}t^2 \\ 0 & 0 & 1 & t \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

check our answer:

```
> expNprime := map(diff, expN, t);
```

$$expNprime := \begin{bmatrix} 0 & 1 & t & \frac{1}{2}t^2 \\ 0 & 0 & 1 & t \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

```
> NexpN := evalm(N&*expN);
```

$$NexpN := \begin{bmatrix} 0 & 1 & t & \frac{1}{2}t^2 \\ 0 & 0 & 1 & t \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Clearly equal. It's also clear that the value of expN at t=0 is the idenity.

Problem 3

```
> J := matrix([
> [1, 1, 0, 0, 0, 0],
> [0, 1, 1, 0, 0, 0],
> [0, 0, 1, 0, 0, 0],
> [0, 0, 0, 2, 1, 0],
> [0, 0, 0, 0, 2, 0],
> [0, 0, 0, 0, 0, 3]]);
```

$$J := \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 \end{bmatrix}$$

S is the diagonal

```
> S:= diag(1, 1, 1, 2, 2, 3);
```

$$S := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 \end{bmatrix}$$

```
> N := evalm(J-S);
```

$$N := \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Check that N is nilpotent:

```
> evalm(N^2); evalm(N^3);
```

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Verifty S and N commute

```
> evalm(S&*N - N&*S);
```

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Find exponential of N:

```
> id6 := diag(1,1,1,1,1,1);
```

$$id6 := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

```
> expN := evalm(id6 + t*N +(t^2/2!)*N^2);
```

$$expN := \begin{bmatrix} 1 & t & \frac{1}{2}t^2 & 0 & 0 & 0 \\ 0 & 1 & t & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & t & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Find exponential of S

```
> evalm(S);
```

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 \end{bmatrix}$$

```
> expS := diag(exp(t), exp(t), exp(t), exp(2*t), exp(2*t),
> exp(3*t));
```

$$expS := \begin{bmatrix} e^t & 0 & 0 & 0 & 0 & 0 \\ 0 & e^t & 0 & 0 & 0 & 0 \\ 0 & 0 & e^t & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{(2\,t)} & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{(2\,t)} & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{(3\,t)} \end{bmatrix}$$

Find exponential of J

```
> expJ := evalm(expS&*expN);
```

$$expJ := \begin{bmatrix} e^t & e^t\,t & \frac{1}{2}\,e^t\,t^2 & 0 & 0 & 0 \\ 0 & e^t & e^t\,t & 0 & 0 & 0 \\ 0 & 0 & e^t & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{(2\,t)} & e^{(2\,t)}\,t & 0 \\ 0 & 0 & 0 & 0 & e^{(2\,t)} & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{(3\,t)} \end{bmatrix}$$

check this answer.

```
> simplify(map2(subs, t=0, expJ));
```

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

```
> expJprime := map(diff, expJ, t);
```

$$expJprime := \begin{bmatrix} e^t & e^t\,t + e^t & \frac{1}{2}\,e^t\,t^2 + e^t\,t & 0 & 0 & 0 \\ 0 & e^t & e^t\,t + e^t & 0 & 0 & 0 \\ 0 & 0 & e^t & 0 & 0 & 0 \\ 0 & 0 & 0 & 2\,e^{(2\,t)} & 2\,e^{(2\,t)}\,t + e^{(2\,t)} & 0 \\ 0 & 0 & 0 & 0 & 2\,e^{(2\,t)} & 0 \\ 0 & 0 & 0 & 0 & 0 & 3\,e^{(3\,t)} \end{bmatrix}$$

```
> JexpJ := evalm(J&*expJ);
```

$$JexpJ := \begin{bmatrix} e^t & e^t\,t + e^t & \frac{1}{2}\,e^t\,t^2 + e^t\,t & 0 & 0 & 0 \\ 0 & e^t & e^t\,t + e^t & 0 & 0 & 0 \\ 0 & 0 & e^t & 0 & 0 & 0 \\ 0 & 0 & 0 & 2\,e^{(2\,t)} & 2\,e^{(2\,t)}\,t + e^{(2\,t)} & 0 \\ 0 & 0 & 0 & 0 & 2\,e^{(2\,t)} & 0 \\ 0 & 0 & 0 & 0 & 0 & 3\,e^{(3\,t)} \end{bmatrix}$$

```
> evalm(expJprime - JexpJ);
```

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Problem 4

```
> A := matrix([
> [568, -347, -31, 1257, -1079, -215],
> [791, -485, -61, 1767, -1504, -312],
> [-681, 419,  45, -1517, 1299,  262],
> [-620, 393,  37, -1393, 1209,  229],
> [-686, 432,  46, -1542, 1331,  259],
> [129,  -60,  -6,  261,  -204,  -59]]);
```

$$A := \begin{bmatrix} 568 & -347 & -31 & 1257 & -1079 & -215 \\ 791 & -485 & -61 & 1767 & -1504 & -312 \\ -681 & 419 & 45 & -1517 & 1299 & 262 \\ -620 & 393 & 37 & -1393 & 1209 & 229 \\ -686 & 432 & 46 & -1542 & 1331 & 259 \\ 129 & -60 & -6 & 261 & -204 & -59 \end{bmatrix}$$

Find the eigenvalues

```
> eigenvals(A);
```

$$-1,\ 1,\ 1,\ 2,\ 2,\ 2$$

Find a basis for the generalized eigenspace for -1

```
> ges1 := nullspace(evalm((A+id6)^6));
```

$$ges1 := \{\left[\frac{-17}{21}, \frac{-22}{21}, \frac{20}{21}, 1, \frac{22}{21}, 0\right]\}$$

```
> v1 := evalm(21*vector([-17/21, -22/21, 20/21, 1, 22/21, 0]));
```

$$v1 := [-17, -22, 20, 21, 22, 0]$$

Basis of the generalized eigenspace for 1

```
> nullspace(evalm((A - id6)^6));
```

$$\{\left[\frac{-4}{3}, \frac{-11}{3}, 2, 0, 1, \frac{-26}{9}\right], \left[\frac{1}{3}, \frac{8}{3}, -1, 1, 0, \frac{23}{9}\right]\}$$

```
> v2 := evalm(9*vector([1/3, 8/3, -1, 1, 0, 23/9]));
```

$$v2 := [3, 24, -9, 9, 0, 23]$$

```
> v3 := evalm(9*vector([-4/3, -11/3, 2, 0, 1, -26/9]));
```

$$v3 := [-12, -33, 18, 0, 9, -26]$$

Basis for the generalized eigenspace for 2

```
> nullspace(evalm((A-2*id6)^6));
```

$$\left\{\left[\frac{-107}{79}, \frac{155}{79}, 0, 1, \frac{-14}{79}, 0\right], \left[\frac{227}{237}, \frac{-85}{79}, 0, 0, \frac{51}{79}, 1\right], \left[\frac{46}{79}, \frac{-249}{79}, 1, 0, \frac{102}{79}, 0\right]\right\}$$

```
> v4 := evalm(237*vector([227/237, -85/79, 0, 0, 51/79, 1]));
```

$$v4 := [227, -255, 0, 0, 153, 237]$$

```
> v5 := evalm(79*vector([-107/79, 155/79, 0, 1, -14/79, 0]));
```

$$v5 := [-107, 155, 0, 79, -14, 0]$$

```
> v6 := evalm(79*vector([46/79, -249/79, 1, 0, 102/79, 0]));
```

$$v6 := [46, -249, 79, 0, 102, 0]$$

Put these vectors in as columns of a matrix P

```
> P := augment(v1, v2, v3, v4, v5, v6);
```

$$P := \begin{bmatrix} -17 & 3 & -12 & 227 & -107 & 46 \\ -22 & 24 & -33 & -255 & 155 & -249 \\ 20 & -9 & 18 & 0 & 0 & 79 \\ 21 & 9 & 0 & 0 & 79 & 0 \\ 22 & 0 & 9 & 153 & -14 & 102 \\ 0 & 23 & -26 & 237 & 0 & 0 \end{bmatrix}$$

```
> B := evalm(inverse(P)&*A&*P);
```

$$B := \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-31}{9} & \frac{64}{9} & 0 & 0 & 0 \\ 0 & \frac{-25}{9} & \frac{49}{9} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{-204}{79} & \frac{124}{79} & \frac{-136}{79} \\ 0 & 0 & 0 & \frac{-1705}{79} & \frac{282}{79} & \frac{-136}{79} \\ 0 & 0 & 0 & \frac{-591}{79} & \frac{-217}{79} & \frac{396}{79} \end{bmatrix}$$

B is block diagonal, with one block for each generalized eigenspace.,

In the basis given by the columns of P, S is diagonal, with the eigenvalues on the diagonal. Call the matrix of S in this basis T.

```
> T := diag(-1, 1, 1, 2, 2, 2);
```

$$T := \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$

M = B-T should be nilpotent.

```
> M := evalm(B-T);
```

$$M := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-40}{9} & \frac{64}{9} & 0 & 0 & 0 \\ 0 & \frac{-25}{9} & \frac{40}{9} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{-362}{79} & \frac{124}{79} & \frac{-136}{79} \\ 0 & 0 & 0 & \frac{-1705}{79} & \frac{124}{79} & \frac{-136}{79} \\ 0 & 0 & 0 & \frac{-591}{79} & \frac{-217}{79} & \frac{238}{79} \end{bmatrix}$$

```
> evalm(M^2);
```

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{6154}{79} & \frac{-2108}{79} & \frac{2312}{79} \\ 0 & 0 & 0 & \frac{5611}{79} & \frac{-1922}{79} & \frac{2108}{79} \end{bmatrix}$$

```
> evalm(M^3);
```

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Now change these matrices back to the standard basis

```
> evalm(A - P&*B&*inverse(P));
```

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

```
> S := evalm(P&*T &*inverse(P));
```

$$S := \begin{bmatrix} -103 & -121 & -195 & 78 & -97 & 33 \\ 63 & -220 & -198 & 447 & -417 & -30 \\ 57 & 164 & 212 & -207 & 212 & -15 \\ 201 & 125 & 255 & 32 & 17 & -69 \\ 129 & 158 & 246 & -111 & 139 & -42 \\ 141 & -48 & 30 & 249 & -204 & -53 \end{bmatrix}$$

```
> N := evalm(A - S);
```

$$N := \begin{bmatrix} 671 & -226 & 164 & 1179 & -982 & -248 \\ 728 & -265 & 137 & 1320 & -1087 & -282 \\ -738 & 255 & -167 & -1310 & 1087 & 277 \\ -821 & 268 & -218 & -1425 & 1192 & 298 \\ -815 & 274 & -200 & -1431 & 1192 & 301 \\ -12 & -12 & -36 & 12 & 0 & -6 \end{bmatrix}$$

```
> evalm(N^2);
```

$$\begin{bmatrix} 28 & -56 & 0 & 140 & -168 & 0 \\ 31 & -62 & 0 & 155 & -186 & 0 \\ -31 & 62 & 0 & -155 & 186 & 0 \\ -34 & 68 & 0 & -170 & 204 & 0 \\ -34 & 68 & 0 & -170 & 204 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

```
> evalm(N^3);
```

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

```
> evalm(S&*N-N&*S);
```

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Compute exp of S, by diagonalization method

```
> evalm(T);
```

$$\begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$

```
> expT := diag(exp(-t), exp(t), exp(t), exp(2*t), exp(2*t),
> exp(2*t));
```

$$expT := \begin{bmatrix} e^{(-t)} & 0 & 0 & 0 & 0 & 0 \\ 0 & e^{t} & 0 & 0 & 0 & 0 \\ 0 & 0 & e^{t} & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{(2\,t)} & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{(2\,t)} & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{(2\,t)} \end{bmatrix}$$

```
> expS := evalm(P&*expT&*inverse(P));
```

$$
\begin{aligned}
&expS := \\
&[105\,e^{t} - 104\,e^{(2\,t)},\ 51\,e^{(-t)} - 32\,e^{t} - 19\,e^{(2\,t)},\ 51\,e^{(-t)} + 42\,e^{t} - 93\,e^{(2\,t)}, \\
&-85\,e^{(-t)} + 177\,e^{t} - 92\,e^{(2\,t)},\ 85\,e^{(-t)} - 158\,e^{t} + 73\,e^{(2\,t)},\ -33\,e^{t} + 33\,e^{(2\,t)}] \\
&[-63\,e^{t} + 63\,e^{(2\,t)},\ 66\,e^{(-t)} + 24\,e^{t} - 89\,e^{(2\,t)},\ 66\,e^{(-t)} - 66\,e^{(2\,t)}, \\
&-110\,e^{(-t)} - 117\,e^{t} + 227\,e^{(2\,t)},\ 110\,e^{(-t)} + 87\,e^{t} - 197\,e^{(2\,t)},\ 30\,e^{t} - 30\,e^{(2\,t)}] \\
&[-57\,e^{t} + 57\,e^{(2\,t)},\ -60\,e^{(-t)} + 16\,e^{t} + 44\,e^{(2\,t)},\ -60\,e^{(-t)} - 30\,e^{t} + 91\,e^{(2\,t)}, \\
&100\,e^{(-t)} - 93\,e^{t} - 7\,e^{(2\,t)},\ -100\,e^{(-t)} + 88\,e^{t} + 12\,e^{(2\,t)},\ 15\,e^{t} - 15\,e^{(2\,t)}] \\
&[-201\,e^{t} + 201\,e^{(2\,t)},\ -63\,e^{(-t)} + 64\,e^{t} - e^{(2\,t)},\ -63\,e^{(-t)} - 66\,e^{t} + 129\,e^{(2\,t)}, \\
&105\,e^{(-t)} - 345\,e^{t} + 241\,e^{(2\,t)},\ -105\,e^{(-t)} + 298\,e^{t} - 193\,e^{(2\,t)},\ 69\,e^{t} - 69\,e^{(2\,t)}] \\
&[-129\,e^{t} + 129\,e^{(2\,t)},\ -66\,e^{(-t)} + 40\,e^{t} + 26\,e^{(2\,t)},\ -66\,e^{(-t)} - 48\,e^{t} + 114\,e^{(2\,t)}, \\
&110\,e^{(-t)} - 219\,e^{t} + 109\,e^{(2\,t)},\ -110\,e^{(-t)} + 193\,e^{t} - 82\,e^{(2\,t)},\ 42\,e^{t} - 42\,e^{(2\,t)}] \\
&[-141\,e^{t} + 141\,e^{(2\,t)},\ 48\,e^{t} - 48\,e^{(2\,t)},\ -30\,e^{t} + 30\,e^{(2\,t)},\ -249\,e^{t} + 249\,e^{(2\,t)}, \\
&204\,e^{t} - 204\,e^{(2\,t)},\ 55\,e^{t} - 54\,e^{(2\,t)}]
\end{aligned}
$$

Find exp of N, nilpotent method

```
> expN := evalm(id6 + t*N + (t^2/2!)*N^2);
```

$$
expN := \begin{bmatrix}
1 + 671\,t + 14\,t^2,\ -226\,t - 28\,t^2,\ 164\,t,\ 1179\,t + 70\,t^2,\ -982\,t - 84\,t^2,\ -248\,t \\
728\,t + \frac{31}{2}\,t^2,\ 1 - 265\,t - 31\,t^2,\ 137\,t,\ 1320\,t + \frac{155}{2}\,t^2,\ -1087\,t - 93\,t^2,\ -282\,t \\
-738\,t - \frac{31}{2}\,t^2,\ 255\,t + 31\,t^2,\ 1 - 167\,t,\ -1310\,t - \frac{155}{2}\,t^2,\ 1087\,t + 93\,t^2,\ 277\,t \\
-821\,t - 17\,t^2,\ 268\,t + 34\,t^2,\ -218\,t,\ 1 - 1425\,t - 85\,t^2,\ 1192\,t + 102\,t^2,\ 298\,t \\
-815\,t - 17\,t^2,\ 274\,t + 34\,t^2,\ -200\,t,\ -1431\,t - 85\,t^2,\ 1 + 1192\,t + 102\,t^2,\ 301\,t \\
-12\,t,\ -12\,t,\ -36\,t,\ 12\,t,\ 0,\ 1 - 6\,t
\end{bmatrix}
$$

Compute the exponential of A

```
> expA := evalm(expS&*expN);
```

$$
\begin{aligned}
&expA := \\
&[\%45\,\%15 + \%44\,\%14 + \%43\,\%13 + \%42\,(-821\,t - 17\,t^2) + \%41\,(-815\,t - \\
&\quad - 12\,\%40\,t, \%45\,(-226\,t - 28\,t^2) + \%44\,\%12 + \%43\,(255\,t + 31\,t^2) \\
&\quad + \%42\,(268\,t + 34\,t^2) + \%41\,(274\,t + 34\,t^2) - 12\,\%40\,t, \\
&164\,\%45\,t + 137\,\%44\,t + \%43\,(1 - 167\,t) - 218\,\%42\,t - 200\,\%41\,t - 36\,\%40 \\
&\%45\,(1179\,t + 70\,t^2) + \%44\,\%11 + \%43\,\%10 + \%42\,\%9 + \%41\,(-1431\,t - \\
&\quad + 12\,\%40\,t, \%45\,(-982\,t - 84\,t^2) + \%44\,(-1087\,t - 93\,t^2) + \%43\,(1087\,t + \\
&\quad + \%42\,\%8 + \%41\,\%7, \\
&-248\,\%45\,t - 282\,\%44\,t + 277\,\%43\,t + 298\,\%42\,t + 301\,\%41\,t + \%40\,(1 - 6 \\
&[\%39\,\%15 + \%38\,\%14 + \%37\,\%13 + \%36\,(-821\,t - 17\,t^2) + \%35\,(-815\,t - \\
&\quad - 12\,\%34\,t, \%39\,(-226\,t - 28\,t^2) + \%38\,\%12 + \%37\,(255\,t + 31\,t^2) \\
&\quad + \%36\,(268\,t + 34\,t^2) + \%35\,(274\,t + 34\,t^2) - 12\,\%34\,t, \\
&164\,\%39\,t + 137\,\%38\,t + \%37\,(1 - 167\,t) - 218\,\%36\,t - 200\,\%35\,t - 36\,\%34 \\
&\%39\,(1179\,t + 70\,t^2) + \%38\,\%11 + \%37\,\%10 + \%36\,\%9 + \%35\,(-1431\,t - \\
&\quad + 12\,\%34\,t, \%39\,(-982\,t - 84\,t^2) + \%38\,(-1087\,t - 93\,t^2) + \%37\,(1087\,t + \\
&\quad + \%36\,\%8 + \%35\,\%7, \\
&-248\,\%39\,t - 282\,\%38\,t + 277\,\%37\,t + 298\,\%36\,t + 301\,\%35\,t + \%34\,(1 - 6 \\
&[\%33\,\%15 + \%32\,\%14 + \%31\,\%13 + \%30\,(-821\,t - 17\,t^2) + \%29\,(-815\,t - \\
&\quad - 12\,\%28\,t, \%33\,(-226\,t - 28\,t^2) + \%32\,\%12 + \%31\,(255\,t + 31\,t^2) \\
&\quad + \%30\,(268\,t + 34\,t^2) + \%29\,(274\,t + 34\,t^2) - 12\,\%28\,t, \\
&164\,\%33\,t + 137\,\%32\,t + \%31\,(1 - 167\,t) - 218\,\%30\,t - 200\,\%29\,t - 36\,\%28 \\
&\%33\,(1179\,t + 70\,t^2) + \%32\,\%11 + \%31\,\%10 + \%30\,\%9 + \%29\,(-1431\,t - \\
&\quad + 12\,\%28\,t, \%33\,(-982\,t - 84\,t^2) + \%32\,(-1087\,t - 93\,t^2) + \%31\,(1087\,t + \\
&\quad + \%30\,\%8 + \%29\,\%7, \\
&-248\,\%33\,t - 282\,\%32\,t + 277\,\%31\,t + 298\,\%30\,t + 301\,\%29\,t + \%28\,(1 - 6 \\
&[\%27\,\%15 + \%26\,\%14 + \%25\,\%13 + \%24\,(-821\,t - 17\,t^2) + \%23\,(-815\,t - \\
&\quad - 12\,\%22\,t, \%27\,(-226\,t - 28\,t^2) + \%26\,\%12 + \%25\,(255\,t + 31\,t^2) \\
&\quad + \%24\,(268\,t + 34\,t^2) + \%23\,(274\,t + 34\,t^2) - 12\,\%22\,t, \\
&164\,\%27\,t + 137\,\%26\,t + \%25\,(1 - 167\,t) - 218\,\%24\,t - 200\,\%23\,t - 36\,\%22 \\
&\%27\,(1179\,t + 70\,t^2) + \%26\,\%11 + \%25\,\%10 + \%24\,\%9 + \%23\,(-1431\,t - \\
&\quad + 12\,\%22\,t, \%27\,(-982\,t - 84\,t^2) + \%26\,(-1087\,t - 93\,t^2) + \%25\,(1087\,t + \\
&\quad + \%24\,\%8 + \%23\,\%7, \\
&-248\,\%27\,t - 282\,\%26\,t + 277\,\%25\,t + 298\,\%24\,t + 301\,\%23\,t + \%22\,(1 - 6 \\
&[\%21\,\%15 + \%20\,\%14 + \%19\,\%13 + \%18\,(-821\,t - 17\,t^2) + \%17\,(-815\,t - \\
&\quad - 12\,\%16\,t, \%21\,(-226\,t - 28\,t^2) + \%20\,\%12 + \%19\,(255\,t + 31\,t^2) \\
&\quad + \%18\,(268\,t + 34\,t^2) + \%17\,(274\,t + 34\,t^2) - 12\,\%16\,t, \\
&164\,\%21\,t + 137\,\%20\,t + \%19\,(1 - 167\,t) - 218\,\%18\,t - 200\,\%17\,t - 36\,\%16 \\
&\%21\,(1179\,t + 70\,t^2) + \%20\,\%11 + \%19\,\%10 + \%18\,\%9 + \%17\,(-1431\,t - \\
&\quad + 12\,\%16\,t, \%21\,(-982\,t - 84\,t^2) + \%20\,(-1087\,t - 93\,t^2) + \%19\,(1087\,t + \\
&\quad + \%18\,\%8 + \%17\,\%7, \\
&-248\,\%21\,t - 282\,\%20\,t + 277\,\%19\,t + 298\,\%18\,t + 301\,\%17\,t + \%16\,(1 - 6 \\
&[\%6\,\%15 + \%5\,\%14 + \%4\,\%13 + \%3\,(-821\,t - 17\,t^2) + \%2\,(-815\,t - 17\,t^2 \\
&\quad - 12\,\%1\,t, \%6\,(-226\,t - 28\,t^2) + \%5\,\%12 + \%4\,(255\,t + 31\,t^2) + \%3\,(268\,t \\
&\quad + \%2\,(274\,t + 34\,t^2) - 12\,\%1\,t, \\
&164\,\%6\,t + 137\,\%5\,t + \%4\,(1 - 167\,t) - 218\,\%3\,t - 200\,\%2\,t - 36\,\%1\,t\,, \\
&\%6\,(1179\,t + 70\,t^2) + \%5\,\%11 + \%4\,\%10 + \%3\,\%9 + \%2\,(-1431\,t - 85\,t^2) \\
&\quad + 12\,\%1\,t, \\
&\%6\,(-982\,t - 84\,t^2) + \%5\,(-1087\,t - 93\,t^2) + \%4\,(1087\,t + 93\,t^2) + \%3\,\%8 \\
&, -248\,\%6\,t - 282\,\%5\,t + 277\,\%4\,t + 298\,\%3\,t + 301\,\%2\,t + \%1\,(1 - 6\,t)]
\end{aligned}
$$

```
> expA := map(simplify, expA);
```

$$
\begin{aligned}
&expA := \\
&[105\,e^{t} - 104\,e^{(2\,t)} + 12\,e^{t}\,t + 659\,e^{(2\,t)}\,t + 14\,e^{(2\,t)}\,t^{2}\,, \\
&51\,e^{(-t)} - 28\,e^{(2\,t)}\,t^{2} - 226\,e^{(2\,t)}\,t - 32\,e^{t} - 19\,e^{(2\,t)}\,, \\
&24\,e^{t}\,t + 140\,e^{(2\,t)}\,t + 51\,e^{(-t)} + 42\,e^{t} - 93\,e^{(2\,t)}\,, \\
&-85\,e^{(-t)} + 70\,e^{(2\,t)}\,t^{2} + 1167\,e^{(2\,t)}\,t + 177\,e^{t} + 12\,e^{t}\,t - 92\,e^{(2\,t)}\,, \\
&85\,e^{(-t)} - 84\,e^{(2\,t)}\,t^{2} - 958\,e^{(2\,t)}\,t - 158\,e^{t} - 24\,e^{t}\,t + 73\,e^{(2\,t)}\,, \\
&4\,e^{t}\,t - 252\,e^{(2\,t)}\,t - 33\,e^{t} + 33\,e^{(2\,t)}] \\
&\left[\frac{31}{2}\,e^{(2\,t)}\,t^{2} + 737\,e^{(2\,t)}\,t - 63\,e^{t} - 9\,e^{t}\,t + 63\,e^{(2\,t)}\,, \right. \\
&66\,e^{(-t)} - 31\,e^{(2\,t)}\,t^{2} - 265\,e^{(2\,t)}\,t + 24\,e^{t} - 89\,e^{(2\,t)}\,, \\
&-18\,e^{t}\,t + 155\,e^{(2\,t)}\,t + 66\,e^{(-t)} - 66\,e^{(2\,t)}\,, \\
&-110\,e^{(-t)} + \frac{155}{2}\,e^{(2\,t)}\,t^{2} + 1329\,e^{(2\,t)}\,t - 117\,e^{t} - 9\,e^{t}\,t + 227\,e^{(2\,t)}\,, \\
&110\,e^{(-t)} - 93\,e^{(2\,t)}\,t^{2} - 1105\,e^{(2\,t)}\,t + 87\,e^{t} + 18\,e^{t}\,t - 197\,e^{(2\,t)}\,, \\
&\left.-3\,e^{t}\,t - 279\,e^{(2\,t)}\,t + 30\,e^{t} - 30\,e^{(2\,t)}\right] \\
&\left[-\frac{31}{2}\,e^{(2\,t)}\,t^{2} - 732\,e^{(2\,t)}\,t - 57\,e^{t} - 6\,e^{t}\,t + 57\,e^{(2\,t)}\,, \right. \\
&-60\,e^{(-t)} + 31\,e^{(2\,t)}\,t^{2} + 255\,e^{(2\,t)}\,t + 16\,e^{t} + 44\,e^{(2\,t)}\,, \\
&-12\,e^{t}\,t - 155\,e^{(2\,t)}\,t - 60\,e^{(-t)} - 30\,e^{t} + 91\,e^{(2\,t)}\,, \\
&100\,e^{(-t)} - \frac{155}{2}\,e^{(2\,t)}\,t^{2} - 1304\,e^{(2\,t)}\,t - 93\,e^{t} - 6\,e^{t}\,t - 7\,e^{(2\,t)}\,, \\
&-100\,e^{(-t)} + 88\,e^{t} + 12\,e^{(2\,t)} + 12\,e^{t}\,t + 1075\,e^{(2\,t)}\,t + 93\,e^{(2\,t)}\,t^{2}\,, \\
&\left.-2\,e^{t}\,t + 279\,e^{(2\,t)}\,t + 15\,e^{t} - 15\,e^{(2\,t)}\right] \\
&[-17\,e^{(2\,t)}\,t^{2} - 797\,e^{(2\,t)}\,t - 201\,e^{t} - 24\,e^{t}\,t + 201\,e^{(2\,t)}\,, \\
&-63\,e^{(-t)} + 34\,e^{(2\,t)}\,t^{2} + 268\,e^{(2\,t)}\,t + 64\,e^{t} - e^{(2\,t)}\,, \\
&-48\,e^{t}\,t - 170\,e^{(2\,t)}\,t - 63\,e^{(-t)} - 66\,e^{t} + 129\,e^{(2\,t)}\,, \\
&105\,e^{(-t)} - 85\,e^{(2\,t)}\,t^{2} - 1401\,e^{(2\,t)}\,t - 345\,e^{t} - 24\,e^{t}\,t + 241\,e^{(2\,t)}\,, \\
&-105\,e^{(-t)} + 102\,e^{(2\,t)}\,t^{2} + 1144\,e^{(2\,t)}\,t + 298\,e^{t} + 48\,e^{t}\,t - 193\,e^{(2\,t)}\,, \\
&-8\,e^{t}\,t + 306\,e^{(2\,t)}\,t + 69\,e^{t} - 69\,e^{(2\,t)}] \\
&[-17\,e^{(2\,t)}\,t^{2} - 800\,e^{(2\,t)}\,t - 129\,e^{t} - 15\,e^{t}\,t + 129\,e^{(2\,t)}\,, \\
&-66\,e^{(-t)} + 34\,e^{(2\,t)}\,t^{2} + 274\,e^{(2\,t)}\,t + 40\,e^{t} + 26\,e^{(2\,t)}\,, \\
&-30\,e^{t}\,t - 170\,e^{(2\,t)}\,t - 66\,e^{(-t)} - 48\,e^{t} + 114\,e^{(2\,t)}\,, \\
&110\,e^{(-t)} - 85\,e^{(2\,t)}\,t^{2} - 1416\,e^{(2\,t)}\,t - 219\,e^{t} - 15\,e^{t}\,t + 109\,e^{(2\,t)}\,, \\
&-110\,e^{(-t)} + 102\,e^{(2\,t)}\,t^{2} + 1162\,e^{(2\,t)}\,t + 193\,e^{t} + 30\,e^{t}\,t - 82\,e^{(2\,t)}\,, \\
&-5\,e^{t}\,t + 306\,e^{(2\,t)}\,t + 42\,e^{t} - 42\,e^{(2\,t)}] \\
&[-141\,e^{t} - 18\,e^{t}\,t + 141\,e^{(2\,t)} + 6\,e^{(2\,t)}\,t\,,\ -12\,e^{(2\,t)}\,t + 48\,e^{t} - 48\,e^{(2\,t)}\,, \\
&-36\,e^{t}\,t - 30\,e^{t} + 30\,e^{(2\,t)}\,,\ -18\,e^{t}\,t + 30\,e^{(2\,t)}\,t - 249\,e^{t} + 249\,e^{(2\,t)}\,, \\
&36\,e^{t}\,t - 36\,e^{(2\,t)}\,t + 204\,e^{t} - 204\,e^{(2\,t)}\,,\ -6\,e^{t}\,t + 55\,e^{t} - 54\,e^{(2\,t)}]
\end{aligned}
$$

check our answer

```
> map2(subs, t=0, expA);
```

$$\begin{bmatrix} e^0 & 0 & 0 & 0 & 0 & 0 \\ 0 & e^0 & 0 & 0 & 0 & 0 \\ 0 & 0 & e^0 & 0 & 0 & 0 \\ 0 & 0 & 0 & e^0 & 0 & 0 \\ 0 & 0 & 0 & 0 & e^0 & 0 \\ 0 & 0 & 0 & 0 & 0 & e^0 \end{bmatrix}$$

```
> expAprime := map(diff, expA, t);
```

$$expAprime :=$$
$$[117\,e^{t}+451\,e^{(2\,t)}+12\,e^{t}\,t+1346\,e^{(2\,t)}\,t+28\,e^{(2\,t)}\,t^{2},$$
$$-51\,e^{(-t)}-56\,e^{(2\,t)}\,t^{2}-508\,e^{(2\,t)}\,t-264\,e^{(2\,t)}-32\,e^{t},$$
$$24\,e^{t}\,t+66\,e^{t}+280\,e^{(2\,t)}\,t-46\,e^{(2\,t)}-51\,e^{(-t)},$$
$$85\,e^{(-t)}+140\,e^{(2\,t)}\,t^{2}+2474\,e^{(2\,t)}\,t+983\,e^{(2\,t)}+189\,e^{t}+12\,e^{t}\,t,$$
$$-85\,e^{(-t)}-168\,e^{(2\,t)}\,t^{2}-2084\,e^{(2\,t)}\,t-812\,e^{(2\,t)}-182\,e^{t}-24\,e^{t}\,t,$$
$$4\,e^{t}\,t-29\,e^{t}-504\,e^{(2\,t)}\,t-186\,e^{(2\,t)}]$$
$$[31\,e^{(2\,t)}\,t^{2}+1505\,e^{(2\,t)}\,t+863\,e^{(2\,t)}-72\,e^{t}-9\,e^{t}\,t,$$
$$-66\,e^{(-t)}-62\,e^{(2\,t)}\,t^{2}-592\,e^{(2\,t)}\,t-443\,e^{(2\,t)}+24\,e^{t},$$
$$-18\,e^{t}\,t-18\,e^{t}+310\,e^{(2\,t)}\,t+23\,e^{(2\,t)}-66\,e^{(-t)},$$
$$110\,e^{(-t)}+155\,e^{(2\,t)}\,t^{2}+2813\,e^{(2\,t)}\,t+1783\,e^{(2\,t)}-126\,e^{t}-9\,e^{t}\,t,$$
$$-110\,e^{(-t)}-186\,e^{(2\,t)}\,t^{2}-2396\,e^{(2\,t)}\,t-1499\,e^{(2\,t)}+105\,e^{t}+18\,e^{t}\,t,$$
$$-3\,e^{t}\,t+27\,e^{t}-558\,e^{(2\,t)}\,t-339\,e^{(2\,t)}]$$
$$[-31\,e^{(2\,t)}\,t^{2}-1495\,e^{(2\,t)}\,t-618\,e^{(2\,t)}-63\,e^{t}-6\,e^{t}\,t,$$
$$60\,e^{(-t)}+62\,e^{(2\,t)}\,t^{2}+572\,e^{(2\,t)}\,t+343\,e^{(2\,t)}+16\,e^{t},$$
$$-12\,e^{t}\,t-42\,e^{t}-310\,e^{(2\,t)}\,t+27\,e^{(2\,t)}+60\,e^{(-t)},$$
$$-100\,e^{(-t)}-155\,e^{(2\,t)}\,t^{2}-2763\,e^{(2\,t)}\,t-1318\,e^{(2\,t)}-99\,e^{t}-6\,e^{t}\,t,$$
$$100\,e^{(-t)}+100\,e^{t}+1099\,e^{(2\,t)}+12\,e^{t}\,t+2336\,e^{(2\,t)}\,t+186\,e^{(2\,t)}\,t^{2},$$
$$-2\,e^{t}\,t+13\,e^{t}+558\,e^{(2\,t)}\,t+249\,e^{(2\,t)}]$$
$$[-34\,e^{(2\,t)}\,t^{2}-1628\,e^{(2\,t)}\,t-395\,e^{(2\,t)}-225\,e^{t}-24\,e^{t}\,t,$$
$$63\,e^{(-t)}+68\,e^{(2\,t)}\,t^{2}+604\,e^{(2\,t)}\,t+266\,e^{(2\,t)}+64\,e^{t},$$
$$-48\,e^{t}\,t-114\,e^{t}-340\,e^{(2\,t)}\,t+88\,e^{(2\,t)}+63\,e^{(-t)},$$
$$-105\,e^{(-t)}-170\,e^{(2\,t)}\,t^{2}-2972\,e^{(2\,t)}\,t-919\,e^{(2\,t)}-369\,e^{t}-24\,e^{t}\,t,$$
$$105\,e^{(-t)}+204\,e^{(2\,t)}\,t^{2}+2492\,e^{(2\,t)}\,t+758\,e^{(2\,t)}+346\,e^{t}+48\,e^{t}\,t,$$
$$-8\,e^{t}\,t+61\,e^{t}+612\,e^{(2\,t)}\,t+168\,e^{(2\,t)}]$$
$$[-34\,e^{(2\,t)}\,t^{2}-1634\,e^{(2\,t)}\,t-542\,e^{(2\,t)}-144\,e^{t}-15\,e^{t}\,t,$$
$$66\,e^{(-t)}+68\,e^{(2\,t)}\,t^{2}+616\,e^{(2\,t)}\,t+326\,e^{(2\,t)}+40\,e^{t},$$
$$-30\,e^{t}\,t-78\,e^{t}-340\,e^{(2\,t)}\,t+58\,e^{(2\,t)}+66\,e^{(-t)},$$
$$-110\,e^{(-t)}-170\,e^{(2\,t)}\,t^{2}-3002\,e^{(2\,t)}\,t-1198\,e^{(2\,t)}-234\,e^{t}-15\,e^{t}\,t,$$
$$110\,e^{(-t)}+204\,e^{(2\,t)}\,t^{2}+2528\,e^{(2\,t)}\,t+998\,e^{(2\,t)}+223\,e^{t}+30\,e^{t}\,t,$$
$$-5\,e^{t}\,t+37\,e^{t}+612\,e^{(2\,t)}\,t+222\,e^{(2\,t)}]$$
$$[-159\,e^{t}-18\,e^{t}\,t+288\,e^{(2\,t)}+12\,e^{(2\,t)}\,t,\ -24\,e^{(2\,t)}\,t-108\,e^{(2\,t)}+48\,e^{t},$$
$$-36\,e^{t}\,t-66\,e^{t}+60\,e^{(2\,t)},\ -18\,e^{t}\,t-267\,e^{t}+60\,e^{(2\,t)}\,t+528\,e^{(2\,t)},$$
$$36\,e^{t}\,t+240\,e^{t}-72\,e^{(2\,t)}\,t-444\,e^{(2\,t)},\ -6\,e^{t}\,t+49\,e^{t}-108\,e^{(2\,t)}]$$

```
> AexpA := evalm(A&*expA);
```

$$AexpA :=$$
$$[117\,e^{t} + 451\,e^{(2\,t)} + 12\,e^{t}\,t + 1346\,e^{(2\,t)}\,t + 28\,e^{(2\,t)}\,t^{2},$$
$$-51\,e^{(-t)} - 56\,e^{(2\,t)}\,t^{2} - 508\,e^{(2\,t)}\,t - 264\,e^{(2\,t)} - 32\,e^{t},$$
$$24\,e^{t}\,t + 66\,e^{t} + 280\,e^{(2\,t)}\,t - 46\,e^{(2\,t)} - 51\,e^{(-t)},$$
$$85\,e^{(-t)} + 140\,e^{(2\,t)}\,t^{2} + 2474\,e^{(2\,t)}\,t + 983\,e^{(2\,t)} + 189\,e^{t} + 12\,e^{t}\,t,$$
$$-85\,e^{(-t)} - 168\,e^{(2\,t)}\,t^{2} - 2084\,e^{(2\,t)}\,t - 812\,e^{(2\,t)} - 182\,e^{t} - 24\,e^{t}\,t,$$
$$4\,e^{t}\,t - 29\,e^{t} - 504\,e^{(2\,t)}\,t - 186\,e^{(2\,t)}]$$
$$[31\,e^{(2\,t)}\,t^{2} + 1505\,e^{(2\,t)}\,t + 863\,e^{(2\,t)} - 72\,e^{t} - 9\,e^{t}\,t,$$
$$-66\,e^{(-t)} - 62\,e^{(2\,t)}\,t^{2} - 592\,e^{(2\,t)}\,t - 443\,e^{(2\,t)} + 24\,e^{t},$$
$$-18\,e^{t}\,t - 18\,e^{t} + 310\,e^{(2\,t)}\,t + 23\,e^{(2\,t)} - 66\,e^{(-t)},$$
$$110\,e^{(-t)} + 155\,e^{(2\,t)}\,t^{2} + 2813\,e^{(2\,t)}\,t + 1783\,e^{(2\,t)} - 126\,e^{t} - 9\,e^{t}\,t,$$
$$-110\,e^{(-t)} - 186\,e^{(2\,t)}\,t^{2} - 2396\,e^{(2\,t)}\,t - 1499\,e^{(2\,t)} + 105\,e^{t} + 18\,e^{t}\,t,$$
$$-3\,e^{t}\,t + 27\,e^{t} - 558\,e^{(2\,t)}\,t - 339\,e^{(2\,t)}]$$
$$[-31\,e^{(2\,t)}\,t^{2} - 1495\,e^{(2\,t)}\,t - 618\,e^{(2\,t)} - 63\,e^{t} - 6\,e^{t}\,t,$$
$$60\,e^{(-t)} + 62\,e^{(2\,t)}\,t^{2} + 572\,e^{(2\,t)}\,t + 343\,e^{(2\,t)} + 16\,e^{t},$$
$$-12\,e^{t}\,t - 42\,e^{t} - 310\,e^{(2\,t)}\,t + 27\,e^{(2\,t)} + 60\,e^{(-t)},$$
$$-100\,e^{(-t)} - 155\,e^{(2\,t)}\,t^{2} - 2763\,e^{(2\,t)}\,t - 1318\,e^{(2\,t)} - 99\,e^{t} - 6\,e^{t}\,t,$$
$$100\,e^{(-t)} + 100\,e^{t} + 1099\,e^{(2\,t)} + 12\,e^{t}\,t + 2336\,e^{(2\,t)}\,t + 186\,e^{(2\,t)}\,t^{2},$$
$$-2\,e^{t}\,t + 13\,e^{t} + 558\,e^{(2\,t)}\,t + 249\,e^{(2\,t)}]$$
$$[-34\,e^{(2\,t)}\,t^{2} - 1628\,e^{(2\,t)}\,t - 395\,e^{(2\,t)} - 225\,e^{t} - 24\,e^{t}\,t,$$
$$63\,e^{(-t)} + 68\,e^{(2\,t)}\,t^{2} + 604\,e^{(2\,t)}\,t + 266\,e^{(2\,t)} + 64\,e^{t},$$
$$-48\,e^{t}\,t - 114\,e^{t} - 340\,e^{(2\,t)}\,t + 88\,e^{(2\,t)} + 63\,e^{(-t)},$$
$$-105\,e^{(-t)} - 170\,e^{(2\,t)}\,t^{2} - 2972\,e^{(2\,t)}\,t - 919\,e^{(2\,t)} - 369\,e^{t} - 24\,e^{t}\,t,$$
$$105\,e^{(-t)} + 204\,e^{(2\,t)}\,t^{2} + 2492\,e^{(2\,t)}\,t + 758\,e^{(2\,t)} + 346\,e^{t} + 48\,e^{t}\,t,$$
$$-8\,e^{t}\,t + 61\,e^{t} + 612\,e^{(2\,t)}\,t + 168\,e^{(2\,t)}]$$
$$[-34\,e^{(2\,t)}\,t^{2} - 1634\,e^{(2\,t)}\,t - 542\,e^{(2\,t)} - 144\,e^{t} - 15\,e^{t}\,t,$$
$$66\,e^{(-t)} + 68\,e^{(2\,t)}\,t^{2} + 616\,e^{(2\,t)}\,t + 326\,e^{(2\,t)} + 40\,e^{t},$$
$$-30\,e^{t}\,t - 78\,e^{t} - 340\,e^{(2\,t)}\,t + 58\,e^{(2\,t)} + 66\,e^{(-t)},$$
$$-110\,e^{(-t)} - 170\,e^{(2\,t)}\,t^{2} - 3002\,e^{(2\,t)}\,t - 1198\,e^{(2\,t)} - 234\,e^{t} - 15\,e^{t}\,t,$$
$$110\,e^{(-t)} + 204\,e^{(2\,t)}\,t^{2} + 2528\,e^{(2\,t)}\,t + 998\,e^{(2\,t)} + 223\,e^{t} + 30\,e^{t}\,t,$$
$$-5\,e^{t}\,t + 37\,e^{t} + 612\,e^{(2\,t)}\,t + 222\,e^{(2\,t)}]$$
$$[-159\,e^{t} - 18\,e^{t}\,t + 288\,e^{(2\,t)} + 12\,e^{(2\,t)}\,t,\ -24\,e^{(2\,t)}\,t - 108\,e^{(2\,t)} + 48\,e^{t},$$
$$-36\,e^{t}\,t - 66\,e^{t} + 60\,e^{(2\,t)},\ -18\,e^{t}\,t - 267\,e^{t} + 60\,e^{(2\,t)}\,t + 528\,e^{(2\,t)},$$
$$36\,e^{t}\,t + 240\,e^{t} - 72\,e^{(2\,t)}\,t - 444\,e^{(2\,t)},\ -6\,e^{t}\,t + 49\,e^{t} - 108\,e^{(2\,t)}]$$

```
> evalm(expAprime- AexpA);
```

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Problem 5

Part A.

```
> A := matrix([
> [5, -1, 0],
> [12, -2, 0],
> [3, -1, 2]]);
```

$$A := \begin{bmatrix} 5 & -1 & 0 \\ 12 & -2 & 0 \\ 3 & -1 & 2 \end{bmatrix}$$

```
> p := charpoly(A, lambda);
```

$$p := \lambda^3 - 5\,\lambda^2 + 8\,\lambda - 4$$

```
> factor(p);
```

$$(\lambda - 1)\,(\lambda - 2)^2$$

Thus, the row of fundemental solutions is

```
> F := vector([exp(t), exp(2*t), t*exp(2*t)]);
```

$$F := [e^t,\, e^{(2\,t)},\, e^{(2\,t)}\,t]$$

```
> w2 := map(diff, F, t);
```

$$w2 := [e^t,\, 2\,e^{(2\,t)},\, 2\,e^{(2\,t)}\,t + e^{(2\,t)}]$$

```
> w3 := map(diff, w2, t);
```

$$w3 := [e^t,\, 4\,e^{(2\,t)},\, 4\,e^{(2\,t)}\,t + 4\,e^{(2\,t)}]$$

Here's the Wronskian Matrix

```
> M := stackmatrix(F, w2, w3);
```

$$M := \begin{bmatrix} e^t & e^{(2\,t)} & e^{(2\,t)}\,t \\ e^t & 2\,e^{(2\,t)} & 2\,e^{(2\,t)}\,t + e^{(2\,t)} \\ e^t & 4\,e^{(2\,t)} & 4\,e^{(2\,t)}\,t + 4\,e^{(2\,t)} \end{bmatrix}$$

Now find the value of the Wronskian matrix at 0

```
> M0:=map2(subs, t=0, M);
```

$$M0 := \begin{bmatrix} e^0 & e^0 & 0 \\ e^0 & 2\,e^0 & e^0 \\ e^0 & 4\,e^0 & 4\,e^0 \end{bmatrix}$$

```
> M0 := simplify(M0);
```

$$M0 := \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 1 & 4 & 4 \end{bmatrix}$$

Now find the vector of prinicpal solutions

```
> r := evalm(F&*inverse(M0));
```

$$r := [4\,e^{t} - 3\,e^{(2\,t)} + 2\,e^{(2\,t)}\,t,\ -4\,e^{t} + 4\,e^{(2\,t)} - 3\,e^{(2\,t)}\,t,\ e^{t} - e^{(2\,t)} + e^{(2\,t)}\,t]$$

Apply Lenonards formula

```
> expA := evalm(r[1]*diag(1,1,1) + r[2]*A + r[3]*A^2);
```

$$expA := \begin{bmatrix} -3\,e^{t} + 4\,e^{(2\,t)} & e^{t} - e^{(2\,t)} & 0 \\ -12\,e^{t} + 12\,e^{(2\,t)} & 4\,e^{t} - 3\,e^{(2\,t)} & 0 \\ -3\,e^{t} + 3\,e^{(2\,t)} & e^{t} - e^{(2\,t)} & e^{(2\,t)} \end{bmatrix}$$

Check our work:

```
> map2(subs, t=0, expA);
```

$$\begin{bmatrix} e^{0} & 0 & 0 \\ 0 & e^{0} & 0 \\ 0 & 0 & e^{0} \end{bmatrix}$$

```
> evalm(map(diff, expA,t)-A&*expA);
```

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Part B.

```
> A := matrix([
> [1, 1, -2],
> [-3, 5, -6],
> [0, 1, -1]]);
```

$$A := \begin{bmatrix} 1 & 1 & -2 \\ -3 & 5 & -6 \\ 0 & 1 & -1 \end{bmatrix}$$

```
> p := charpoly(A, lambda);
```

$$p := \lambda^{3} - 5\,\lambda^{2} + 8\,\lambda - 4$$

```
> factor(p);
```

$$(\lambda - 1)\,(\lambda - 2)^{2}$$

The row of fundamental solutions is the same as in the last part:

```
> F = evalm(F);
```

$$F = [e^{t},\ e^{(2\,t)},\ e^{(2\,t)}\,t]$$

The Wronskian matrix at 0, and the row r of principal solutions are the same

```
> r = evalm(r);
```

$$r = [4\,e^{t} - 3\,e^{(2\,t)} + 2\,e^{(2\,t)}\,t,\ -4\,e^{t} + 4\,e^{(2\,t)} - 3\,e^{(2\,t)}\,t,\ e^{t} - e^{(2\,t)} + e^{(2\,t)}\,t]$$

Now apply Leonard's formula

```
> expA := evalm(r[1]*diag(1,1,1)+r[2]*A + r[3]*A^2);
```

$$expA := \begin{bmatrix} 3\,e^{(2\,t)} - 3\,e^{(2\,t)}\,t - 2\,e^{t} & e^{(2\,t)}\,t & 2\,e^{t} - 2\,e^{(2\,t)} \\ -6\,e^{t} + 6\,e^{(2\,t)} - 9\,e^{(2\,t)}\,t & e^{(2\,t)} + 3\,e^{(2\,t)}\,t & 6\,e^{t} - 6\,e^{(2\,t)} \\ -3\,e^{t} + 3\,e^{(2\,t)} - 3\,e^{(2\,t)}\,t & e^{(2\,t)}\,t & 3\,e^{t} - 2\,e^{(2\,t)} \end{bmatrix}$$

check our work

```
> map2(subs, t=0, expA);
```

$$\begin{bmatrix} e^{0} & 0 & 0 \\ 0 & e^{0} & 0 \\ 0 & 0 & e^{0} \end{bmatrix}$$

```
> evalm(map(diff, expA,t)-A&*expA);
```

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Part C.

Same as part A. Oops!

Part D.

```
> A := matrix([
> [-79, -4, -36, 76],
> [614, 7, 325, -558],
> [262, 4, 137, -240],
> [71, -2, 43, -61]]);
```

$$A := \begin{bmatrix} -79 & -4 & -36 & 76 \\ 614 & 7 & 325 & -558 \\ 262 & 4 & 137 & -240 \\ 71 & -2 & 43 & -61 \end{bmatrix}$$

```
> p := charpoly(A, lambda);
```

$$p := \lambda^4 - 4\,\lambda^3 + 14\,\lambda^2 - 20\,\lambda + 25$$

```
> factor(p);
```

$$(\lambda^2 - 2\,\lambda + 5)^2$$

```
> solve((lambda^2-2*lambda+5)=0, lambda);
```

$$1 + 2\,I,\ 1 - 2\,I$$

So, the roots are 1+2I, 1-2I, each with multiplicity 2
The row of fundemental solutions is

```
> F := vector([exp(t)*cos(2*t), exp(t)*sin(2*t), t*exp(t)*cos(2*t),
> t*exp(t)*sin(2*t)]);
```

$$F := [e^{t}\cos(2\,t),\ e^{t}\sin(2\,t),\ t\,e^{t}\cos(2\,t),\ t\,e^{t}\sin(2\,t)]$$

construct the Wronskian matrix

```
> w2 := map(diff, F, t);
```

$w2 := [e^t \cos(2\,t) - 2\,e^t \sin(2\,t),\ e^t \sin(2\,t) + 2\,e^t \cos(2\,t),$
$e^t \cos(2\,t) + t\,e^t \cos(2\,t) - 2\,t\,e^t \sin(2\,t),\ e^t \sin(2\,t) + t\,e^t \sin(2\,t) + 2\,t\,e^t \cos(2\,t)]$

```
> w3 := map(diff, w2, t);
```

$w3 := [-3\,e^t \cos(2\,t) - 4\,e^t \sin(2\,t),\ -3\,e^t \sin(2\,t) + 4\,e^t \cos(2\,t),$
$2\,e^t \cos(2\,t) - 4\,e^t \sin(2\,t) - 3\,t\,e^t \cos(2\,t) - 4\,t\,e^t \sin(2\,t),$
$2\,e^t \sin(2\,t) + 4\,e^t \cos(2\,t) - 3\,t\,e^t \sin(2\,t) + 4\,t\,e^t \cos(2\,t)]$

```
> w4 := map(diff,w3,t);
```

$w4 := [-11\,e^t \cos(2\,t) + 2\,e^t \sin(2\,t),\ -11\,e^t \sin(2\,t) - 2\,e^t \cos(2\,t),$
$-9\,e^t \cos(2\,t) - 12\,e^t \sin(2\,t) - 11\,t\,e^t \cos(2\,t) + 2\,t\,e^t \sin(2\,t),$
$-9\,e^t \sin(2\,t) + 12\,e^t \cos(2\,t) - 11\,t\,e^t \sin(2\,t) - 2\,t\,e^t \cos(2\,t)]$

```
> M := stackmatrix(F,w2,w3,w4);
```

$M :=$
$[\%2\,,\ \%1\,,\ t\,e^t \cos(2\,t)\,,\ t\,e^t \sin(2\,t)]$
$[\%2 - 2\,e^t \sin(2\,t)\,,\ \%1 + 2\,e^t \cos(2\,t)\,,\ \%2 + t\,e^t \cos(2\,t) - 2\,t\,e^t \sin(2\,t)\,,$
$\%1 + t\,e^t \sin(2\,t) + 2\,t\,e^t \cos(2\,t)]$
$[-3\,\%2 - 4\,e^t \sin(2\,t)\,,\ -3\,\%1 + 4\,e^t \cos(2\,t)\,,$
$2\,\%2 - 4\,e^t \sin(2\,t) - 3\,t\,e^t \cos(2\,t) - 4\,t\,e^t \sin(2\,t)\,,$
$2\,\%1 + 4\,e^t \cos(2\,t) - 3\,t\,e^t \sin(2\,t) + 4\,t\,e^t \cos(2\,t)]$
$[-11\,\%2 + 2\,e^t \sin(2\,t)\,,\ -11\,\%1 - 2\,e^t \cos(2\,t)\,,$
$-9\,\%2 - 12\,e^t \sin(2\,t) - 11\,t\,e^t \cos(2\,t) + 2\,t\,e^t \sin(2\,t)\,,$
$-9\,\%1 + 12\,e^t \cos(2\,t) - 11\,t\,e^t \sin(2\,t) - 2\,t\,e^t \cos(2\,t)]$
$\%1 := e^t \sin(2\,t)$
$\%2 := e^t \cos(2\,t)$

Value of the wronskian matrix at 0 is

```
> M0 := map2(subs, t=0, M);
```

$M0 :=$
$[e^0 \cos(0)\,,\ e^0 \sin(0)\,,\ 0\,,\ 0]$
$[e^0 \cos(0) - 2\,e^0 \sin(0)\,,\ e^0 \sin(0) + 2\,e^0 \cos(0)\,,\ e^0 \cos(0)\,,\ e^0 \sin(0)]$
$[-3\,e^0 \cos(0) - 4\,e^0 \sin(0)\,,\ -3\,e^0 \sin(0) + 4\,e^0 \cos(0)\,,\ 2\,e^0 \cos(0) - 4\,e^0 \sin(0)\,,$
$2\,e^0 \sin(0) + 4\,e^0 \cos(0)]$
$[-11\,e^0 \cos(0) + 2\,e^0 \sin(0)\,,\ -11\,e^0 \sin(0) - 2\,e^0 \cos(0)\,,\ -9\,e^0 \cos(0) - 12\,e^0 \sin(0)\,,$
$-9\,e^0 \sin(0) + 12\,e^0 \cos(0)]$

```
> M0 := simplify(M0);
```

$$M0 := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ -3 & 4 & 2 & 4 \\ -11 & -2 & -9 & 12 \end{bmatrix}$$

So, the row of principal solutions is

```
> r := evalm(F&*inverse(M0));
```

$$r := \left[e^t\cos(2\,t) - \frac{13}{16}\,\%1 + \frac{5}{8}\,t\,e^t\cos(2\,t) + \frac{5}{4}\,t\,e^t\sin(2\,t),\ \frac{15}{16}\,\%1 - \frac{7}{8}\,t\,e^t\cos(2\,t) - \frac{1}{2}\,t\,e^t\sin(2\,t),\right.$$
$$\left. -\frac{3}{16}\,\%1 + \frac{3}{8}\,t\,e^t\cos(2\,t) + \frac{1}{4}\,t\,e^t\sin(2\,t),\ \frac{1}{16}\,\%1 - \frac{1}{8}\,t\,e^t\cos(2\,t)\right]$$

$$\%1 := e^t\sin(2\,t)$$

Apply Leonard's formula

```
> expA := evalm(r[1]*diag(1,1,1,1)+r[2]*A +r[3]*A^2 + r[4]*A^3);
```

$$expA :=$$
$$[\%1 - 31\,e^t\sin(2\,t) - 18\,t\,e^t\cos(2\,t) - 22\,t\,e^t\sin(2\,t)\,,\ -2\,\%2\,,$$
$$-13\,\%2 - 10\,t\,e^t\cos(2\,t) - 12\,t\,e^t\sin(2\,t)\,,\ 30\,\%2 + 16\,t\,e^t\cos(2\,t) + 20\,t\,e^t\sin(2\,t)]$$
$$[288\,\%2 + 38\,t\,e^t\cos(2\,t) + 24\,t\,e^t\sin(2\,t)\,,\ \%1 + 3\,e^t\sin(2\,t)\,,$$
$$152\,\%2 + 21\,t\,e^t\cos(2\,t) + 13\,t\,e^t\sin(2\,t)\,,\ -262\,\%2 - 34\,t\,e^t\cos(2\,t) - 22\,t\,e^t\sin(2\,t)$$
$$]$$
$$[122\,\%2 + 18\,t\,e^t\cos(2\,t) + 22\,t\,e^t\sin(2\,t)\,,\ 2\,\%2\,,$$
$$\%1 + 63\,e^t\sin(2\,t) + 10\,t\,e^t\cos(2\,t) + 12\,t\,e^t\sin(2\,t)\,,$$
$$-112\,\%2 - 20\,t\,e^t\sin(2\,t) - 16\,t\,e^t\cos(2\,t)]$$
$$[40\,\%2 - 9\,t\,e^t\cos(2\,t) - 11\,t\,e^t\sin(2\,t)\,,\ -\%2\,,\ 24\,\%2 - 5\,t\,e^t\cos(2\,t) - 6\,t\,e^t\sin(2\,t)$$
$$,\ \%1 - 35\,e^t\sin(2\,t) + 8\,t\,e^t\cos(2\,t) + 10\,t\,e^t\sin(2\,t)]$$
$$\%1 := e^t\cos(2\,t)$$
$$\%2 := e^t\sin(2\,t)$$

check our work

```
> simplify(map2(subs, t=0, expA));
```

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

```
> simplify(evalm(map(diff, expA,t)-A&*expA));
```

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Problem 6

Will have to be done by pencil and paper.

Problem 7

The solution is omitted.

Problem 8