

50 pts.

**Problem 1.** In each part, you are given the augmented matrix of a system of linear equations, with the coefficient matrix in row echelon form. Determine if the system is consistent and, if so, use the method of backsubstitution to find all solutions.

A.

$$\left[ \begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 1 & 7 \end{array} \right]$$

B.

$$\left[ \begin{array}{ccccc|c} 1 & -2 & 1 & 1 & -5 & 1 \\ 0 & 1 & 3 & 2 & -1 & 3 \\ 0 & 0 & 0 & 1 & 2 & 5 \\ 0 & 0 & 0 & 0 & 0 & 7 \end{array} \right]$$

C.

$$\left[ \begin{array}{ccccc|c} 1 & -2 & 1 & 1 & -5 & 9 \\ 0 & 1 & 3 & 2 & -1 & 11 \\ 0 & 0 & 0 & 1 & 2 & 6 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

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50 pts.

**Problem 2.** In each part you are given the augmented matrix of a system of linear equations, with the coefficient matrix in reduced row echelon form. Determine if the system is consistent and, if it is consistent, find all solutions.

A.

$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

B.

$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 \end{array} \right]$$

C.

$$\left[ \begin{array}{ccccc|c} 1 & 0 & -1 & 2 & 0 & -1 \\ 0 & 1 & -3 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

40 pts.

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**Problem 3.** In each part, solve the linear system. Show the augmented matrix you start with and the augmented matrix you finish with. It's not necessary to show individual row operations, you can just find the [Reduced] Row Echelon Form with your calculator.

A.

$$\begin{aligned}4x + 2y + z &= 9 \\2x + 2y &= 2 \\2x + 2y + z &= 5\end{aligned}$$

B.

$$\begin{aligned}2x + y - z &= 4 \\x + y + z &= 3 \\3x + 2y &= 7\end{aligned}$$

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70 pts.

**Problem 4.** Consider the matrix

$$A = \begin{bmatrix} 1 & 0 & -2 & 1 & 2 & 6 \\ 1 & 1 & -1 & 3 & 6 & 12 \\ 1 & 0 & -2 & 2 & 5 & 7 \\ 5 & -4 & -14 & -2 & -3 & 7 \\ 4 & 3 & -5 & 2 & -4 & 34 \end{bmatrix}.$$

The RREF of  $A$  is the matrix

$$R = \begin{bmatrix} 1 & 0 & -2 & 0 & -1 & 5 \\ 0 & 1 & 1 & 0 & -2 & 4 \\ 0 & 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

- A. Find a basis for the nullspace of  $A$ .
- B. Find a basis for the rowspace of  $A$ .
- C. Find a basis for the columnspace of  $A$ .
- D. What is the rank of  $A$ ?

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45 pts.

**Problem 5.** Consider row operations on matrices with 3 rows.

Recall that for each row operation there is a corresponding elementary matrix  $E$  so that  $EA$  is the same as the matrix obtained by applying the row operation to  $A$ .

- A. Consider the row operation  $R_2 \leftrightarrow R_3$ .
- Find the corresponding elementary matrix  $E$ .
  - Find the inverse row operation.
  - Find the elementary matrix that corresponds to the inverse row operation.
- B. Consider the row operation  $R_3 \leftarrow 3R_3$ .
- Find the corresponding elementary matrix  $E$ .
  - Find the inverse row operation.
  - Find the elementary matrix that corresponds to the inverse row operation.
- C. Consider the row operation  $R_2 \leftarrow R_2 - 3R_1$
- Find the corresponding elementary matrix  $E$ .
  - Find the inverse row operation.
  - Find the elementary matrix that corresponds to the inverse row operation.

40 pts.

**Problem 6.** In each part, use row operations to determine if the matrix  $A$  is invertible and, if so, to find the inverse. It is not necessary to show the individual row operations (you can just use the `rref` key on the calculator). Show the augmented matrix you start with and the augmented matrix you finish with. Give the matrix entries in fractional form.

A.

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & -1 \\ 0 & 2 & 0 \end{bmatrix}$$

B.

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

50 pts.

**Problem 7.** Let  $S$  be the subspace of  $\mathbb{R}^4$  spanned by the vectors

$$\mathbf{v}_1 = \begin{bmatrix} 2 \\ 2 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 3 \\ -1 \\ 3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 8 \\ 0 \\ 5 \\ -3 \end{bmatrix}, \quad \mathbf{v}_4 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}.$$

- A. Cut down the list of vectors above to a basis for  $S$ . What is the dimension of  $S$ ?
- B. For each of the following vectors, determine if the vector is in  $S$  and, if so, express it as a linear combination of the basis vectors you found in the previous part of the problem.

$$\mathbf{w}_1 = \begin{bmatrix} -3 \\ 5 \\ -2 \\ 5 \end{bmatrix}, \quad \mathbf{w}_2 = \begin{bmatrix} 5 \\ 3 \\ 4 \\ 0 \end{bmatrix}$$

60 pts.

**Problem 8.** Pare the following set of vectors down to a basis of  $\mathbb{R}^3$ . Express the vectors that are not in the basis as linear combinations of the basis vectors.

$$\mathbf{v}_1 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{v}_4 = \begin{bmatrix} 10 \\ 7 \\ 4 \end{bmatrix}, \quad \mathbf{v}_5 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$$

70 pts.

**Problem 9.** Consider the matrix

$$A = \begin{bmatrix} 0 & 3 & 1 \\ 1 & 3 & 2 \\ 2 & 1 & 1 \end{bmatrix}$$

- A. Find the cofactors  $A_{12}$  and  $A_{33}$ .
- B. Compute  $\det(A)$ , using the cofactor expansion along a selected row or column.
- C. Find the adjoint matrix  $\text{adj}(A)$  from its definition in terms of cofactors.
- D. Use the information above to find  $A^{-1}$ .

40 pts.

**Problem 10.** Find the following determinant by the method of elimination, i.e., by using row operations and keeping track of the effect of the row operations on the determinant. Show the row operations you use and the intermediate determinants (you use a calculator to perform the row ops).

Sorry, no credit for finding it by another method.

$$\begin{vmatrix} 0 & 2 & 2 & 2 \\ 0 & 2 & 1 & 1 \\ 2 & 2 & 4 & 0 \\ 5 & 5 & 5 & 0 \end{vmatrix}$$

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40 pts.

**Problem 11.** Use Cramer's rule to solve the following system.

$$\begin{array}{rccccrcr} 2x & + & y & + & z & = & 1 \\ & & y & - & 2z & = & 0 \\ 2x & & & + & z & = & 0 \end{array}$$

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40 pts.

**Problem 12.** Let

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 0 \end{bmatrix}.$$

Find the characteristic polynomial of  $A$  and the eigenvalues of  $A$ .

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60 pts.

**Problem 13.** In each part you are given an matrix  $A$  and its eigenvalues. Find a basis for each of the eigenspaces of  $A$ . Determine if  $A$  is diagonalizable, and if it is, find a matrix  $P$  and a diagonal matrix  $D$  so that  $P^{-1}AP = D$ .

A.

$$A = \begin{bmatrix} 0 & 1 & 1 \\ -1 & 3 & 0 \\ -1 & 2 & 1 \end{bmatrix}, \quad \text{Eigenvalues} = 1, 2.$$

B.

$$A = \begin{bmatrix} 8 & 9 & 9 \\ 0 & 2 & 0 \\ -6 & -9 & -7 \end{bmatrix}, \quad \text{Eigenvalues} = -1, 2.$$

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40 pts.

**Problem 14.** Let  $A$  be an  $n \times n$  matrix. If  $A^2 = I$ , what are the possible values of  $\det(A)$ ? Justify your answer.

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# EXAM

Practice Exam # 1

Math 3351, Spring 2003

Feb.10, 2003

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- Write all of your answers on separate sheets of paper. You can keep the exam questions when you leave. You may leave when finished.
- You **must** show enough work to justify your answers. Unless otherwise instructed, give exact answers, not approximations (e.g.,  $\sqrt{2}$ , not 1.414).
- This exam has 14 problems. There are **695 points total**.

Good luck!