

40 pts.

Problem 1. Find the absolute maximum and absolute minimum values of the function $f(x) = x^3 - 6x^2 + 1$ on the interval $[-3, 5]$.

50 pts.

Problem 2. A flower bed is to be constructed. One side of the bed will be formed by the long, straight wall of a building, and the other three sides will be fenced. If there is 8 feet of fencing available, find the dimensions of the flower bed that will give the maximum area.

Be sure to show what interval you are maximizing the function over, and to justify that you have really found the max, not just a critical point.

50 pts.

Problem 3. We want to construct a cylindrical tin can, with a closed top and bottom. The volume of the can is to be 16π cubic inches. What should the dimensions of the can be to minimize the cost of the materials (i.e., to minimize the surface area).

Be sure to show the interval you are minimizing your function over, and be sure to check that you really have the absolute minimum, not just a critical point.

50 pts.

Problem 4. Consider the function $f(x) = x^4 - 4x^3 + 2$.

A. Find all the critical points. Construct the sign table for $f'(x)$. Determine where $f(x)$ is increasing and decreasing. Determine if each critical point is a relative max, a relative min, or neither.

B. Find the sign table for $f''(x)$. Find all inflection points. Determine where the graph $y = f(x)$ is concave up and concave down.

C. Sketch the graph $y = f(x)$, plotting all critical points and inflection points.

60 pts.

Problem 5. Consider the function

$$f(x) = \frac{x^2}{x^2 - 1}.$$

- A. Find all vertical and horizontal asymptotes for the graph $y = f(x)$.
 - B. Find all the critical points. Construct the sign table for $f'(x)$. Determine where $f(x)$ is increasing and decreasing. Determine if each critical point is a relative max, a relative min, or neither.
 - C. Find the sign table for $f''(x)$. Find all inflection points. Determine where the graph $y = f(x)$ is concave up and concave down.
 - D. Sketch the graph $y = f(x)$, plotting all critical points, inflection points, and asymptotes.
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40 pts.

Problem 6. In each part, find the limit.

A.

$$\lim_{x \rightarrow 0} \frac{\sin(3x)}{x + \sin(x)}$$

B.

$$\lim_{x \rightarrow 0} \frac{\sin(x) - x}{x^3}$$

C.

$$\lim_{x \rightarrow \infty} \frac{\ln(x)}{\sqrt{x}}$$

D.

$$\lim_{x \rightarrow 0^+} \sqrt{x} \ln(x)$$

EXAM

Exam #4

Math 1351-007, Fall 2002

April 28, 2003

- This is a **Take-Home Exam**, due May 5.
- Write all of your answers on separate sheets of paper. You can keep the exam questions.
- You **must** show enough work to justify your answers. Unless otherwise instructed, give exact answers, not approximations (e.g., $\sqrt{2}$, not 1.414).
- This exam has 6 problems. There are **290 points total**.

Good luck!