

60 pts.

Problem 1. In each part determine if the series is convergent or divergent. If it is convergent find the sum. (These are geometric or telescoping series.)

A.

$$\sum_{k=1}^{\infty} \frac{(-1)^{k+1} 3}{5^{2k}} = \frac{3}{5^2} - \frac{3}{5^4} + \frac{3}{5^6} - \frac{3}{5^8} + \dots$$

B.

$$\sum_{k=0}^{\infty} 3e^{k/2} = 3 + 3e^{1/2} + 3e^{2/2} + 3e^{3/2} + \dots$$

C.

$$\sum_{k=4}^{\infty} \frac{2}{(2k+1)(2k+3)} = \frac{2}{99} + \frac{2}{143} + \frac{2}{195} + \dots$$

40 pts.

Problem 2. In each part, express the given number as a single fraction.

A. $0.0444\bar{4}$.

B. $1.67351\overline{351}$.

40 pts.

Problem 3. A patient is given an injection of 10 units of a certain drug just before noon each day. The drug is eliminated from the body exponentially, so the amount of drug from one injection that remains at noon t days later is $10e^{-t/3}$.

Find an expression for the amount of the drug present in the patients body at noon on the first few days. If the treatment continues for many days, what limit does the amount of drug present in the patient's body at noon approach?

200 pts.

Problem 4. In each part, determine if the series is convergent or divergent. Be sure to say which test you are using and to show the details of how the test is applied.

Part 1.

$$\sum_{k=1}^{\infty} \frac{n^5}{3^n}.$$

Part 2.

$$\sum_{k=1}^{\infty} \frac{k}{\sqrt{k^2 + 1}}.$$

Part 3.

$$\sum_{k=1}^{\infty} \frac{1}{\sqrt[3]{k^2}}.$$

Part 4.

$$\sum_{k=1}^{\infty} \left(\frac{3k}{5k + 1} \right)^{2k}.$$

Part 5.

$$\sum_{k=3}^{\infty} \frac{\ln(k)}{k}.$$

Part 6.

$$\sum_{k=1}^{\infty} \frac{k}{k^3 - 2k^2 + k + 5}.$$

Part 7.

$$\sum_{k=1}^{\infty} \frac{3^k}{k^2 2^k}.$$

Part 8.

$$\sum_{k=1}^{\infty} \frac{5^{2k+1}}{(2k + 1)!}.$$

Part 9.

$$\sum_{k=1}^{\infty} \frac{1}{\sqrt{k^2 + k + 1}}$$

Part 10.

$$\sum_{k=1}^{\infty} \frac{k^k}{(2k + 1)!}$$

60 pts.

Problem 5. Assume that $x > 0$. In each case, find the set of all $x > 0$ for which the series converges.

A.

$$\sum_{k=1}^{\infty} \frac{x^{2k}}{(2k)!}$$

B.

$$\sum_{k=1}^{\infty} \frac{2^k x^k}{k}$$

C.

$$\sum_{k=1}^{\infty} \frac{x^{2k}}{k^3}$$

EXAM

Exam #3

Math 1352-006, Fall 2003

December 3, 2003

- This is a take-home exam. You may look at the textbook and your notes. You may discuss the problems, but write up the solutions by yourself.
- **This exam is due by 5 p.m. on Monday, December 8, 2003**
- You **must** show enough work to justify your answers. Unless otherwise instructed, give exact answers, not approximations (e.g., $\sqrt{2}$, not 1.414).
- This exam has 5 problems. There are **400 points total**.

Good luck!