70 pts.

Problem 1. In each part, find $f'(x)$.

A.

$$f(x) = (x^2 + 2x + 5)^{10}$$

B.

$$f(x) = \frac{x^3}{x^2 + 1}$$

C.

$$f(x) = e^{x^2+3x+5}$$

D.

$$f(x) = \sec^5(x^3)$$

E.

$$f(x) = \cos^{-1}(x^2)$$

F.

$$f(x) = (x^2 + 3)^2(x^3 + 2x + 1)^4$$

G.

$$f(x) = \ln(\ln(x))$$

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40 pts.

Problem 2. A watering trough is 6 feet long, see Figure 1. The cross section is an isoceles triangle that is 4 feet high and 4 feet wide at the top, see Figure 2.

If water is being poured into the trough at a rate of 3 cubic feet per second, how fast is the water level in the trough rising when the water is 2 feet deep?

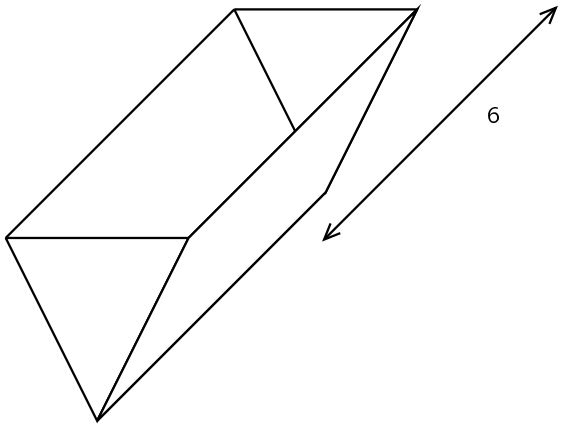

Figure 1. The Watering Trough.

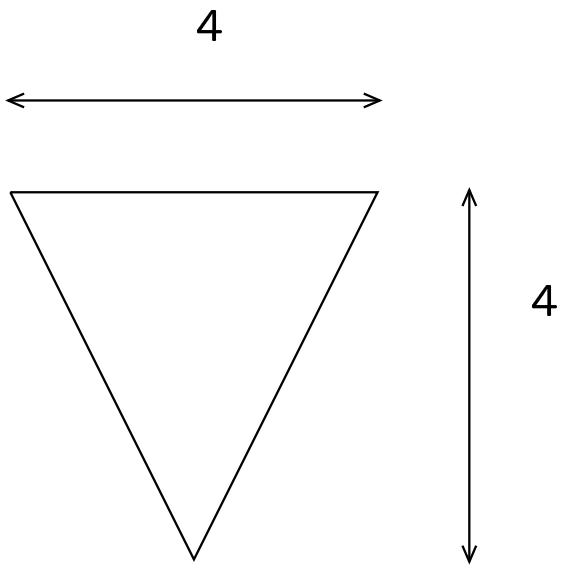

Figure 2. The End of the Trough.

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40 pts.

Problem 3. Consider the curve defined by the equation

$$2x^2 + 2xy + y^2 = 18.$$

Find coordinates of the highest and lowest points on this curve. (See Figure 3. Hint: implicit differentiation.)

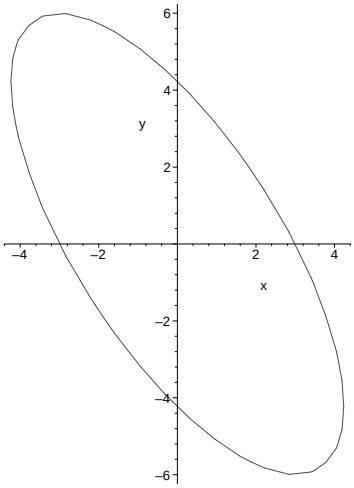

Figure 3. Find the Highest and Lowest Points.

40 pts.

Problem 4. Sketch the graph of the function $f(x) = xe^{-2x^2}$, plotting all relative max's and min's, all critical points, all inflection points and all asymptotes.

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40 pts.

Problem 5. A box is to be constructed with a square bottom and no top. The volume of the box is to be 128 cubic inches. The material that will be used for the bottom of the box costs 10 cents per square inch. The material for the vertical sides of the box costs 20 cents per square inch.

What should the dimensions of the box be to minimize the cost of the materials? Be sure to specify the interval you are minimizing over and be sure to justify that you actually have the minimum, not just a critical point.

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50 pts.

Problem 6. Alice and Bob are in the desert in their SUV, near a long straight road. The closest point on the road, call it point A, is 12 miles away. A gas station is located at point B, 30 miles down the road from point A. The SUV can travel at 36 miles per hour through the desert and at 60 miles per hour on the road. Alice and Bob want to reach the gas station in the least time, so they plan to drive straight through the desert to some point on the road between A and B, and then along the road to the gas station.

A. What point on the road should they head for to reach the gas station in the least time? (Be sure to specify the interval you are minimizing over and to justify that you have the minimum, not just a critical point.)

B. Pat, the mischievous calculus student, who is riding in the back seat, mischievously gives Alice and Bob the directions that will maximize the time to reach the gas station. What directions does Pat give?

EXAM

Take Home Exam

Math 1351-007 and Math 1351-012
Fall 2002

November 25, 2002

- Write all of your answers on separate sheets of paper.
- You **must** show enough work to justify your answers. Unless otherwise instructed, give exact answers, not approximations (e.g., $\sqrt{2}$, not 1.414).
- You may discuss this exam with other students in the course, but write up the solutions by yourself.
- This exam has 6 problems. There are **280 points total**.
- This exam is due at the final exam for the course.

Good luck!