Problem 1. For this problem, refer to the definitions and notation used in the first problem set (on sums of unordered terms).

A. Let X be a set and let $f\colon X \to [0, \infty)$. Show that f is summable if and only if $\sum_X^* f < \infty$ and, in this case,

$$\sum_X f = \sum_X^* f.$$

B. If $f\colon X \to \mathbb{R}$, show that f is summable if and only if f^+ and f^- are summable, and in this case,

$$\sum_X f = \sum_X f^+ - \sum_X f^-.$$

C. Let ν be counting measure on X, i.e., ν is defined on all subsets of X as follows: If $E \subseteq X$ is finite, $\nu(E)$ is the number of elements in E; if E is infinite, $\nu(E) = \infty$.

Let $f\colon X \to [0, \infty]$. Show that

$$\int f \, d\nu = \sum_X^* f.$$

Problem 2. Do the following problems from Section 2.3 (pp. 59ff).

A. Problem 20.

B. Problem 28.

C. Problem 30.

Problem 3. Do Problem 44, Section 2.4, page 64.

Problem 4. Do the following problems from Section 2.5, (pp. 68ff).

1. Problem 48.
2. Problem 49.
3. Problem 50.

Problem 5. Do the following problems from Section 2.6, (pp. 76ff).

1. Problem 55.
2. Problem 59.

Problem Set

Take-home Exam

Math 5322, Fall 2001

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Good luck!