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# EXAM

Exam #2

Math 1430, Fall 2001

October 22, 2001

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# ANSWERS



40 pts.

**Problem 1.** Find the equation of the line that passes through the points  $(-2, 4)$  and  $(4, 1)$ . Give your answer in the form  $y = mx + b$ . Find the  $x$ -intercept and  $y$ -intercept of this line.

*Answer:*

Let  $(x_1, y_1) = (-2, 4)$  and  $(x_2, y_2) = (4, 1)$ . The slope of the line is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 4}{4 - (-2)} = \frac{-3}{6} = -\frac{1}{2}.$$

We now know the slope of the line and a point on the line (2 points, in fact) we can use the point-slope form of the equation of a line. The general form is

$$y - y_0 = m(x - x_0).$$

In our case  $m = -1/2$  and we can choose  $(x_0, y_0) = (4, 1)$ . This gives us

$$y - 1 = -\frac{1}{2}(x - 4)$$

as the point-slope form of the equation of our line. We can simplify this as follows

$$y - 1 = -\frac{1}{2}(x - 4)$$

$$y - 1 = -\frac{1}{2}x + \frac{1}{2}4$$

$$y - 1 = -\frac{1}{2}x + 2$$

$$y = -\frac{1}{2}x + 2 + 1$$

$$y = -\frac{1}{2}x + 3.$$

Thus, the equation of the line in the required form is

$$\boxed{y = -\frac{1}{2}x + 3}.$$

Since this is the slope-intercept form, we can immediately read off the  $y$ -intercept

$$\boxed{y\text{-intercept: } y = 3.}$$

To find the  $x$ -intercept, we set  $y = 0$  in the equation of the line and solve for  $x$ . Thus, we solve as follows

$$0 = -\frac{1}{2}x + 3$$

$$\frac{1}{2}x = 3$$

$$x = 6.$$

So, the answer for the  $x$ -intercept is

$$\boxed{x\text{-intercept: } x = 6.}$$

40 pts.

**Problem 2.** Write the quadratic function  $f(x) = -2x^2 + 4x + 6$  in standard form  $f(x) = a(x - h)^2 + k$  by completing the square. Find the vertex of the parabola  $y = f(x)$  and the maximum or minimum value of  $f(x)$ .

*Answer:*

Note the correction in the formula for the function.

Comparing  $f(x) = -2x^2 + 4x + 6$  with the template  $f(x) = ax^2 + bx + c$ , we see that  $a = -2$ ,  $b = 4$  and  $c = 6$ . We know that  $h = -b/(2a)$ , so

$$h = -\frac{4}{2(-2)} = 1.$$

So far we know

$$f(x) = -2x^2 + 4x + 6 = -2(x - 1)^2 + k. \quad (1)$$

To find  $k$ , we work out the algebra as follows

$$\begin{aligned} -2x^2 + 4x + 6 &= -2(x - 1)^2 + k \\ &= -2(x^2 - 2x + 1) + k \\ &= -2x^2 + 4x - 2 + k, \end{aligned}$$

so we must have  $6 = -2 + k$ , or  $k = 8$ . Plugging this into (1), we get  $f(x)$  in standard form as

$$\boxed{f(x) = -2(x - 1)^2 + 8}.$$

From this we can read off the vertex

$$\boxed{\text{vertex: } (1,8)}.$$

Since the leading coefficient  $a = -2$  is negative, the graph of  $f(x)$  is a parabola opening downward. Thus,  $f$  takes its maximum value at the vertex. Thus, the answer to this part of the question is

$$\boxed{\text{maximum value} = 8}.$$

40 pts.

**Problem 3.** Sketch the graph of the rational function

$$f(x) = \frac{x + 1}{x - 2}.$$

Show any vertical or horizontal asymptotes and the  $x$ -intercept and  $y$ -intercept.

*Answer:*

We will have vertical asymptotes at points where the denominator is zero, so in this case we have

$$\boxed{\text{Vertical asymptote: } x = 2} .$$

To find possible horizontal asymptotes, we consider the behavior as  $x \rightarrow \pm\infty$ . Dividing the top and bottom by the highest power of  $x$  that appears, we have

$$\begin{aligned} f(x) &= \frac{x+1}{x-2} \\ &= \frac{(x+1)(1/x)}{(x-2)(1/x)} \\ &= \frac{1 + \frac{1}{x}}{1 - \frac{2}{x}} \\ &\rightarrow \frac{1+0}{1-0} = 1 \end{aligned}$$

since  $1/x \rightarrow 0$  as  $x \rightarrow \pm\infty$ . Thus, we have

$$\boxed{\text{Horizontal asymptote: } y = 1} .$$

To find the  $y$ -intercept, just calculate  $f(0) = 1/(-2)$ , so

$$\boxed{y\text{-intercept: } y = -1/2} .$$

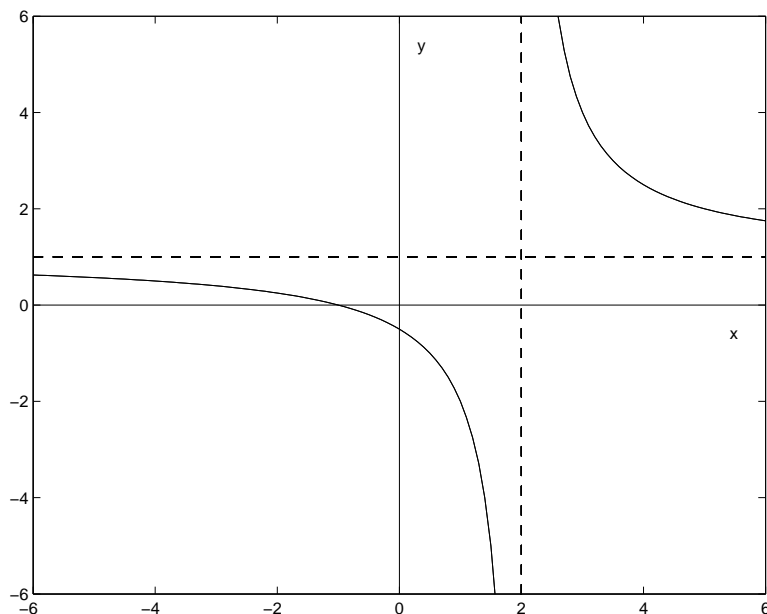
To find the  $x$ -intercepts, set  $y = 0$  and solve for  $x$ . Thus, we have to solve the equation

$$0 = \frac{x+1}{x-2} .$$

The only way for a fraction to be zero is for the top to be zero, so we get

$$\boxed{x\text{-intercept: } x = -1} .$$

It's easier on the computer to plot the graph exactly than to sketch it, so here's a picture of the graph.



If you try to plot this on a calculator, it will look funny near the vertical asymptote, since the calculator is not smart enough to know there is a vertical asymptote.

To determine the behavior of the function near the vertical asymptote, you can reason as follows. If  $x$  is a bit less than 2,  $x - 2$  is negative (and small) and  $x + 1$  is positive (and close to 3). Thus  $(x + 1)/(x - 2)$  is negative and big, so the graph must go down as it approaches the vertical asymptote from the left. Similar reasoning shows that the graph goes up as it approaches the vertical asymptote from the right.

40 pts.

**Problem 4.** In each part evaluate the given logarithm exactly.

A.  $\log_2(1)$ .

*Answer:*

The logarithmic equation  $\log_2(1) = x$  is equivalent to the exponential equation  $2^x = 1$ . Since  $2^0 = 1$ , we conclude  $\log_2(1) = 0$ .

B.  $\log_2(16)$ .

*Answer:*

If  $x = \log_2(16)$ , the equivalent exponential equation is  $2^x = 16$ . Since  $2^4 = 16$ ,  $\log_2(16) = 4$ .

C.  $\log_2(1/8)$ .

*Answer:*

If  $x = \log_2(1/8)$ , the equivalent exponential equation is  $2^x = 1/8$ . We have  $2^{-3} = 1/2^3 = 1/8$ , so  $x = -3$ . Thus,  $\boxed{\log_2(1/8) = -3}$ .

D.  $\log_7(7^{25})$ .

*Answer:*

Recall that one of the basic properties of logarithms is

$$\log_b(b^x) = x.$$

Applying this rule, we have  $\boxed{\log_7(7^{25}) = 25}$ .

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40 pts.

**Problem 5.** Write the following expression in terms of simpler logarithmic forms.

$$\log_b\left(\frac{\sqrt[3]{xy^2}}{z^5}\right)$$

*Answer:*

Proceed as followings, using the indicated properties of logarithms.

$$\begin{aligned}\log_b\left(\frac{\sqrt[3]{xy^2}}{z^5}\right) &= \log_b(\sqrt[3]{xy^2}) - \log_b(z^5) && \text{Quotient Rule} \\ &= \log_b[(xy^2)^{1/3}] - \log_b(z^5) \\ &= \frac{1}{3} \log_b(xy^2) - 5 \log_b(z) && \text{Power Rule} \\ &= \frac{1}{3} [\log_b(x) + \log_b(y^2)] - 5 \log_b(z) && \text{Product Rule} \\ &= \frac{1}{3} \log_b(x) + \frac{1}{3} \log_b(y^2) - 5 \log_b(z) \\ &= \frac{1}{3} \log_b(x) + \frac{1}{3} (2) \log_b(y) - 5 \log_b(z) && \text{Power Rule} \\ &= \boxed{\frac{1}{3} \log_b(x) + \frac{2}{3} \log_b(y) - 5 \log_b(z)}.\end{aligned}$$

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40 pts.

**Problem 6.** Solve the equation

$$3(7^x) = 90$$

for  $x$ . Use your calculator and give the answer rounded to 4 decimal places.

*Answer:*

We want to isolate the unknown exponent  $x$ . First, divide both sides of the equation

$$3(7^x) = 90$$

by 3 to get

$$7^x = 30.$$

Take common logs of both sides, which gives us

$$\log(7^x) = \log(30).$$

Using the power rule for logarithms, the we can rewrite this equation as

$$x \log(7) = \log(30).$$

Dividing both sides by  $\log(7)$  yields

$$x = \frac{\log(30)}{\log(7)},$$

which tells us what  $x$  is. Plugging the right-hand side into a calculator (type  $(\log 30)/(\log 7)$  on a TI-86) gives

$$x \approx 1.7479$$

after rounding to 4 decimal places.

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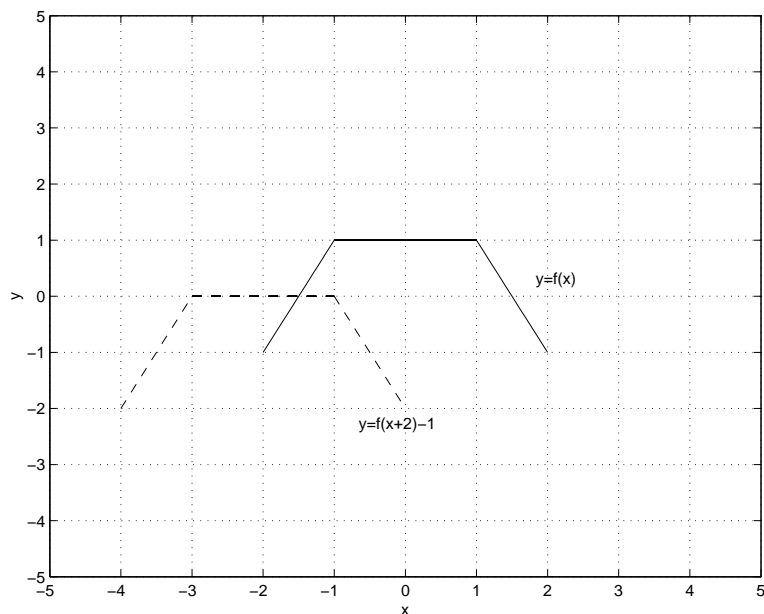
40 pts.

**Problem 7. Turn in this page!**

- A. The graph of  $y = f(x)$  is drawn below. On the same grid, draw the graph  $y = f(x + 2) - 1$ .

*Answer:*

The graph of  $y = f(x + 2) - 1$  is obtained by shifting the graph of  $y = f(x)$  **left** 2 units and down 1 unit. Here is the picture with both graphs on it.



- B. The graph of  $y = f(x)$  is drawn again below. On the same grid, draw the graph of  $y = -2f(x)$ .

*Answer:*

The graph of  $y = -2f(x)$  is obtained by multiplying all the  $y$ -coordinates on the graph  $y = f(x)$  by  $-2$ . Thus, the point  $(-2, -1)$  on the original graph becomes  $(-2, 2)$  on the new graph. The point  $(-1, 1)$  becomes  $(-1, -2)$  on the new graph;  $(1, 1)$  becomes  $(1, -2)$ ; and  $(2, -1)$  becomes  $(2, 2)$ . Here is the picture with both graphs drawn.

