
EXAM

Exam #1

Math 1430, Fall 2001

September 21, 2001

ANSWERS

40 pts.

Problem 1. Perform the indicated operations and simplify.

A. $3(x^2 + 4x - 7) - 2(x^2 - 9x + 5)$

Answer:

$$\begin{aligned} 3(x^2 + 4x - 7) - 2(x^2 - 9x + 5) &= 3x^2 + 12x - 21 - 2x^2 + 18x - 10 \\ &= \boxed{x^2 + 30x - 31}. \end{aligned}$$

B. $(2x - 1)(5x + 3)$

Answer:

$$\begin{aligned} (2x - 1)(5x + 3) &= (2x)(5x) + (2x)(3) + (-1)(5x) + (-1)(3) \quad (\text{FOIL}) \\ &= 10x^2 + 6x - 5x - 3 \\ &= \boxed{10x^2 + x - 3}. \end{aligned}$$

40 pts.

Problem 2. If possible, factor the polynomial $2x^2 - 4x - 6$ using integer coefficients.

Answer:

Comparing with $ax^2 + bx + c$ we see that $a = 2$, $b = -4$ and $c = -6$. Thus, $ac = -12$. We want to factor -12 as pq , with $p + q = b = -4$. This can be done with $p = -6$ and $q = 2$. Then we have

$$\begin{aligned} 2x^2 - 4x - 6 &= 2x^2 + (2 - 6)x - 6 &&= 2x^2 + 2x - 6x - 6 \\ &= 2x(x + 1) - 6(x + 1) \\ &= \boxed{(2x - 6)(x + 1)} \\ &= \boxed{2(x - 3)(x + 1)}. \end{aligned}$$

40 pts.

Problem 3. In each part, perform the indicated operation and reduce to lowest terms.

A.

$$\frac{1}{x} - \frac{1}{x + 1}$$

Answer:

$$\begin{aligned}\frac{1}{x} - \frac{1}{x+1} &= \frac{1(x+1)}{x(x+1)} - \frac{1(x)}{(x+1)(x)} \\ &= \frac{x+1}{x(x+1)} - \frac{x}{x(x+1)} \\ &= \frac{x+1-x}{x(x+1)} \\ &= \boxed{\frac{1}{x(x+1)}}\end{aligned}$$

B.

$$\frac{\frac{x}{x^2-4}}{\frac{x^2}{x+2}}$$

Answer:

$$\begin{aligned}\frac{\frac{x}{x^2-4}}{\frac{x^2}{x+2}} &= \frac{x}{x^2-4} \cdot \frac{x+2}{x^2} \\ &= \frac{x}{(x-2)(x+2)} \cdot \frac{x+2}{x^2} \\ &= \boxed{\frac{1}{x(x-2)}}.\end{aligned}$$

40 pts.

Problem 4. In each part, simplify and express answers using positive exponents only.

A.

$$\frac{5x^{-2}y^{-3}}{10xy^{-5}}$$

Answer:

$$\begin{aligned}\frac{5x^{-2}y^{-3}}{10xy^{-5}} &= \frac{5}{10}x^{-2-1}y^{-3-(-5)} \\ &= \frac{1}{2}x^{-3}y^2 \\ &= \boxed{\frac{y^2}{2x^3}}.\end{aligned}$$

B.

$$(2x^{-2}y^4)^{-3}$$

Answer:

$$\begin{aligned}(2x^{-2}y^4)^{-3} &= 2^{-3}(x^{-2})^{-3}(y^4)^{-3} \\ &= 2^{-3}x^{(-2)(-3)}y^{4(-3)} \\ &= 2^{-3}x^6y^{-12} \\ &= \frac{x^6}{2^3y^{12}} \\ &= \boxed{\frac{x^6}{8y^{12}}}.\end{aligned}$$

40 pts.

Problem 5. In each part, carry out the indicated operations and simplify, express answers using positive exponents only

A.

$$2x^{2/3}(x^{1/3} - 4x^2)$$

Answer:

$$\begin{aligned}2x^{2/3}(x^{1/3} - 4x^2) &= (2x^{2/3})x^{1/3} - (2x^{2/3})(4x^2) \\ &= 2x^{(2/3)+1/3} - 8x^{2/3+2} \\ &= \boxed{2x - 8x^{8/3}}.\end{aligned}$$

B.

$$(8x^3y^{-6})^{1/3}$$

Answer:

$$\begin{aligned}(8x^3y^{-6})^{1/3} &= 8^{1/3}(x^3)^{1/3}(y^{-6})^{1/3} \\ &= \sqrt[3]{8}x^{3(1/3)}y^{-6(1/3)} \\ &= 2xy^{-2} \\ &= \boxed{\frac{2x}{y^2}}.\end{aligned}$$

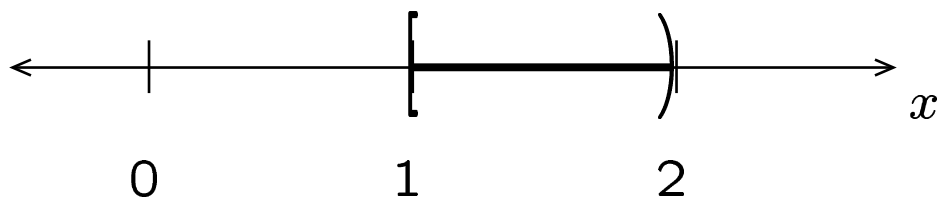


Figure 1: Graph of the Solution Set for Problem 6

40 pts.

Problem 6. Solve the inequality $0 < 4 - 2x \leq 2$. Graph the solution set.

Answer:

We start with the inequality

$$0 < 4 - 2x \leq 2.$$

Adding -4 to each express gives

$$-4 < -2x \leq -2.$$

Now multiply through the inequality by $-1/2$. **Remember that multiplying through by a negative number reverse the inequalities.** Thus, we get

$$(-1/2)(-4) > (-1/2)(-2)x \geq (-1/2)(-2)$$

which simplifies to

$$2 > x \geq 1,$$

which is the same as

$$\boxed{1 \leq x < 2},$$

or, in interval notation, $\boxed{[1, 2)}$. The graph of the solution set is shown in Figure 1.

70 pts.

Problem 7.

A. Solve $2x^2 - 6x = 0$.

Answer:

$$2x^2 - 6x = 2x(x - 3),$$

so the solutions are $\boxed{x = 0}$ and $\boxed{x = 3}$.

B. Solve $2x^2 - x - 1 = 0$.

Answer:

Comparing with $ax^2 + bx + c$ we have $a = 2$, $b = -1$ and $c = -1$. We want to find p and q so that $ac = -2 = pq$ and $b = -1 = p + q$. We can take $p = -2$, and $q = 1$. Then

$$\begin{aligned} 2x^2 - x - 1 &= 2x^2 + (1 - 2)x - 1 \\ &= 2x^2 + x - 2x - 1 \\ &= x(2x + 1) - (2x + 1) \\ &= (2x + 1)(x - 1). \end{aligned}$$

The factor $x - 1$ is zero when $x = 1$. The factor $2x + 1 = 0$ when $2x = -1$ or $x = -1/2$. Thus the solutions are $\boxed{x = 1}$ and $\boxed{x = -1/2}$.

C. **Use the quadratic formula to solve** $4x^2 + 11x - 3 = 0$. Give a factorization of this polynomial with integer coefficients, using the factor theorem.

Answer:

Comparing $4x^2 + 11x - 3$ to $ax^2 + bx + c$, we have $a = 4$, $b = 11$ and $c = -3$. By the quadratic formula, the roots are

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-11 \pm \sqrt{11^2 - 4(4)(-3)}}{2(4)} \\ &= \frac{-11 \pm \sqrt{121 + 48}}{8} \\ &= \frac{-11 \pm \sqrt{169}}{8} \\ &= \frac{-11 \pm 13}{8}. \end{aligned}$$

Thus, the two roots of $4x^2 + 11x - 3 = 0$ are

$$\begin{aligned} r_1 &= \frac{-11 + 13}{8} = 2/8 = \boxed{1/4} \\ r_2 &= \frac{-11 - 13}{8} = \frac{-24}{8} = \boxed{-3}. \end{aligned}$$

According to the factor theorem, we have

$$\begin{aligned} 4x^2 + 11x - 3 &= a(x - r_1)(x - r_2) \\ &= 4(x - 1/4)(x - (-3)) \\ &= \boxed{(4x - 1)(x + 3)}. \end{aligned}$$