1. Suppose that a population of married couples have heights in inches $X$ for the wife and $Y$ for the husband. Suppose that $(X, Y)^T$ has a bivariate normal distribution with parameters $\mu_X = 66, \mu_Y = 70, \sigma_X = 2.5, \sigma_Y = 2.7,$ and $\rho = 3$. What is the probability that the husband is taller than his wife? Does the probability increase or decrease with $\rho$?

2. Let $X_1, \ldots, X_{n_1}$ and let $Y_1, \ldots, Y_{n_2}$ be independent random samples from $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$, respectively. Find the distributions of the following random variables. In each case, name the distribution and give its parameters.

   a. \( \frac{1}{\sigma_1^2} \sum_{i=1}^{n_1} (X_i - \mu_1)^2 \), and \( \frac{1}{\sigma_1^2} \sum_{i=1}^{n_1} (X_i - c_i)^2 \), for arbitrary constants $c_1, \ldots, c_{n_1}$.

   b. \( \frac{n_2}{\sigma_1^2} (\bar{x} - \mu_0)^2 \), \( \frac{n_1}{\sigma_1^2} (\bar{x} - \mu_1)^2 \) ($\mu_0$ here is an arbitrary constant).

   c. \( n_1 (\bar{x} - \mu_1)^2 / \sigma_1^2 + n_2 (\bar{y} - \mu_2)^2 / \sigma_2^2 \).

   d. \( \frac{n_2}{n_1} \frac{[\sum_{i=1}^{n_1} (x_i - \mu_0)^2]}{[\sum_{i=1}^{n_2} (y_i - \bar{y})^2]} \). Here, assume $\sigma_1^2 = \sigma_2^2 = \sigma^2$ and $\mu_0$ is an arbitrary constant.

3. Let $(X_1, Y_1), \ldots, (X_n, Y_n)$ be a random sample from the bivariate normal distribution with parameters $\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho$. Find a constant $K$ so that

\[
T = K \frac{(\bar{X} - \bar{Y}) - \delta}{\left\{ \sum_{i=1}^{n} [(X_i - Y_i) - (\bar{X} - \bar{Y})]^2 \right\}^{1/2}} \sim t(m, \theta).
\]

Express $m$ and $\theta$ as functions of the parameters and the constant $\delta$. (Hint: Let $D_i = X_i - Y_i$. Express $T$ as a function of the $D_i$'s.)

4. Problem \( \text{5.27 in our text.} \)

5. Problem \( \text{5.30 in our text.} \)