Engin. **5.3** Face masks used by firefighters often fail by having their lenses fall out when exposed to very high temperatures. A manufacturer of face masks claims that for their masks the average temperature at which pop out occurs is 550°F. A sample of 75 masks are tested and the average temperature at which the lense popped out was 470°F. Based on this information is the manufacturer's claim valid?

a. Identify the population of interest to us in this problem.

b. Would an answer to the question posed involve estimation or testing a hypothesis?

5.4 Refer to Exercise 5.3. How might you select a sample of face masks from the manufacturer to evaluate the claim?

5.2 Estimation of μ

Engin. **5.5** A company that manufacturers coffee for use in commercial machines monitors the caffeine content in its coffee. The company selects 50 samples of coffee every hour from its production line and determines the caffeine content. From historical data, the caffeine content (in milligrams, mg) is known to have a normal distribution with $\sigma = 7.1$ mg. During a 1-hour time period, the 50 samples yielded a mean caffeine content of $\bar{y} = 110$ mg.

a. Calculate a 95% confidence interval for the mean caffeine content μ of the coffee produced during the hour in which the 50 samples were selected.

b. Explain to the CEO of the company in nonstatistical language, the interpretation of the constructed confidence interval.

5.6 Refer to Exercise 5.5. The engineer in charge of the coffee manufacturing process examines the confidence intervals for the mean caffeine content calculated over the past several weeks and is concerned that the intervals are too wide to be of any practical use. That is, they are not providing a very precise estimate of μ.

a. What would happen to the width of the confidence intervals if the level of confidence of each interval is increased from 95% to 99%?

b. What would happen to the width of the confidence intervals if the number of samples per hour was increased from 50 to 100?

5.7 Refer to Exercise 5.5. Because the company is sampling the coffee production process every hour, there are 720 confidence intervals for the mean caffeine content μ constructed every month.

a. If the level of confidence remains at 95% for the 720 confidence intervals in a given month, how many of the confidence intervals would you expect to fail to contain the value of μ and hence provide an incorrect estimation of the mean caffeine content?

b. If the number of samples is increased from 50 to 100 each hour, how many of the 95% confidence intervals would you expect to fail to contain the value of μ in a given month?

c. If the number of samples remains at 50 each hour but the level of confidence is increased from 95% to 99% for each of the intervals, how many of the 95% confidence intervals would you expect to fail to contain the value of μ in a given month?

Bus. **5.8** As part of the recruitment of new businesses in their city, the economic development department of the city wants to estimate the gross profit margin of small businesses (under one million dollars in sales) currently residing in their city. A random sample of the previous years annual reports of 15 small businesses shows the mean net profit margins to be 7.2% (of sales) with a standard deviation of 12.5%.

a. Construct a 99% confidence interval for the mean gross profit margin of μ of all small businesses in the city.

b. The city manager reads the report and states that the confidence interval for μ constructed in part (a) is not valid because the data are obviously not normally distributed and thus the sample size is too small. Based on just knowing the mean and standard deviation of the sample of 15 businesses, do you think the city manager is valid in his conclusion about the data? Explain your answer.

Soc. **5.9** A social worker is interested in estimating the average length of time spent outside of prison for first offenders who later commit a second crime and are sent to prison again. A random sample of $n = 150$ prison records in the county courthouse indicates that the average length of prison-free life between first and second offenses is 3.2 years, with a standard deviation of 1.1 years. Use the sample information to estimate μ, the mean prison-free life between first and second offenses for all prisoners on record in the county courthouse. Construct a 95% confidence interval for μ. Assume that σ can be replaced by s.

Ag. **5.10** The rust mite, a major pest of citrus in Florida, punctures the cells of leaves and fruit. Damage by rust mites is readily recognizable because the injured fruit displays a brownish (rust) color and is somewhat reduced in size depending on the severity of the attack. If the rust mites are not controlled, the affected groves have a substantial reduction in both the fruit yield and the fruit quality. In either case, the citrus grower suffers financially because the produce is of a lower grade and sells for less on the fresh-fruit market. This year, more and more citrus growers have gone to a program of preventive maintenance spraying for rust mites. In evaluating the effectiveness of the program, a random sample of sixty 10-acre plots, one plot from each of 60 groves, is selected. These show an average yield of 850 boxes of fruit, with a standard deviation of 100 boxes. Give a 95% confidence interval for μ, the average (10-acre) yield for all groves utilizing such a maintenance spraying program. Assume that σ can be replaced by s.

Ag. **5.11** An experiment is conducted to examine the susceptibility of root stocks of a variety of lemon trees to a specific larva. Forty of the plants are subjected to the larvae and examined after a fixed period of time. The response of interest is the logarithm of the number of larvae per gram that is counted on each root stock. For these 40 plants the sample mean is 9.02 and the standard deviation is 1.12. Use these data to construct a 90% confidence interval for μ, the mean susceptibility for the population of lemon tree root stocks from which the sample was drawn. Assume that σ can be replaced by s.

Gov. **5.12** A problem of interest to the United States, other governments, and world councils concerned with the critical shortage of food throughout the world is finding a method to estimate the total amount of grain crops that will be produced throughout the world in a particular year.

One method of predicting total crop yields is based on satellite photographs of Earth's surface. Because a scanning device reads the total acreage of a particular type of grain with error, it is necessary to have the device read many equal-sized plots of a particular planting to calibrate the reading on the scanner with the actual acreage. Satellite photographs of one hundred 50-acre plots of wheat are read by the scanner and give a sample average and standard deviation

$$\bar{y} = 3.27 \quad s = .23$$

Find a 95% confidence interval for the mean scanner reading for the population of all 50-acre plots of wheat. Explain the meaning of this interval.

5.3 Choosing the Sample Size for Estimating μ

5.13 Refer to Example 5.4. Suppose we estimate σ with $\hat{\sigma} = .75$.

a. If the level of confidence remains at 99% but the tolerable width of the interval is .4, how large a sample size is required?

b. If the level of confidence decreases to 95% but the specified width of the interval remains at .5, how large a sample size is required?

c. If the level of confidence increases to 99.5% but the specified width of the interval remains at .5, how large a sample size is required?

5.14 In any given situation, if the level of confidence and the standard deviation are kept constant, how much would you need to increase the sample size to decrease the width of the interval to half its original size?

Bio. **5.15** A biologist wishes to estimate the effect of an antibiotic on the growth of a particular bacterium by examining the mean amount of bacteria present per plate of culture when a fixed amount of the antibiotic is applied. Previous experimentation with the antibiotic on this type of bacteria indicates that the standard deviation of the amount of bacteria present is approximately 13 cm^2. Use this information to determine the number of observations (cultures that must be developed and then tested) to estimate the mean amount of bacteria present, using a 99% confidence interval with a half-width of 3 cm^2.

Soc. **5.16** The city housing department wants to estimate the average rent for rent-controlled apartments. They need to determine the number of renters to include in the survey in order to estimate the average rent to within \$50 using a 95% confidence interval. From past results, the rent for controlled apartments ranged from $200 to $1,500 per month. How many renters are needed in the survey to meet the requirements?

5.17 Refer to Exercise 5.16. Suppose the mayor has reviewed the proposed survey and decides on the following changes:

a. If the level of confidence is increased to 99% with the average rent estimated to within $25, what sample size is required?

b. Suppose the budget for the project will not support both increasing the level of confidence and reducing the width of the interval. Explain to the mayor the impact on the estimation of the average rent of not raising the level of confidence from 95% to 99%.

5.4 **A Statistical Test for μ**

5.18 A researcher designs a study to test the hypotheses H_0: $\mu \geq 28$ versus H_a: $\mu < 28$. A random sample of 50 measurements from the population of interest yields $\bar{y} = 25.9$ and $s = 5.6$.

a. Using $\alpha = .05$, what conclusions can you make about the hypotheses based on the sample information?

b. Calculate the probability of making a Type II error if the actual value of μ is at most 27.

c. Could you have possibly made a Type II error in your decision in part (a)? Explain your answer.

5.19 Refer to Exercise 5.18. Sketch the power curve for rejecting H_0: $\mu \geq 28$ by determining PWR(μ_a) for the following values of μ: 22, 23, 24, 25, 26, and 27.

a. Interpret the power values displayed in your graph.

b. Suppose we keep $n = 50$ but change to $\alpha = .01$. Without actually recalculating the values for PWR(μ_a), sketch on the same graph as your original power curve, the new power curve for $n = 50$ and $\alpha = .01$.

c. Suppose we keep $\alpha = .05$ but change to $n = 20$. Without actually recalculating the values for PWR(μ_a), sketch on the same graph as your original power curve the new power curve for $n = 20$ and $\alpha = .05$.

5.20 Use a computer software program to simulate 100 samples of size 25 from a normal distribution with $\mu = 30$ and $\sigma = 5$. Test the hypotheses H_0: $\mu = 30$ versus H_a: $\mu \neq 30$ using each of the 100 samples of $n = 25$ and using $\alpha = .05$.

a. How many of the 100 tests of hypotheses resulted in your reaching the decision to reject H_0?

b. Suppose you were to conduct 100 tests of hypotheses and in each of these tests the true hypothesis was H_0. On the average, how many of the 100 tests would have resulted in your incorrectly rejecting H_0, if you were using $\alpha = .05$?

c. What type of error are you making if you incorrectly reject H_0?

5.21 Refer to Exercise 5.20. Suppose the population mean was 32 instead of 30. Simulate 100 samples of size $n = 25$ from a normal distribution with $\mu = 32$ and $\sigma = 5$. Using $\alpha = .05$, test the hypotheses H_0: $\mu = 30$ versus H_a: $\mu \neq 30$ using each of the 100 samples of size $n = 25$.

a. What proportion of the 100 tests of hypotheses resulted in the correct decision, that is, reject H_0?

b. In part (a), you were estimating the power of the test when $\mu_a = 32$, that is, the ability of the testing procedure to detect that the null hypothesis was false. Now, calculate the power of your test to detect that $\mu = 32$, that is, compute PWR($\mu_a = 32$).

c. Based on your calculation in (b) how many of the 100 tests of hypotheses would you expect to correctly reject H_0? Compare this value with the results from your simulated data.

5.22 Refer to Exercises 5.20 and 5.21.

a. Answer the questions posed in these exercises with $\alpha = .01$ in place of $\alpha = .05$. You can use the data set simulated in Exercise 5.20, but the exact power of the test, PWR($\mu_a = 32$), must be recalculated.

b. Did decreasing α from .05 to .01 increase or decrease the power of the test? Explain why this change occurred.

Med. **5.23** A study was conducted of 90 adult male patients following a new treatment for congestive heart failure. One of the variables measured on the patients was the increase in exercise capacity (in minutes) over a 4-week treatment period. The previous treatment regime had produced an average increase of $\mu = 2$ minutes. The researchers wanted to evaluate whether the new treatment had increased the value of μ in comparison to the previous treatment. The data yielded $\bar{y} = 2.17$ and $s = 1.05$.

a. Using $\alpha = .05$, what conclusions can you draw about the research hypothesis?

b. What is the probability of making a Type II error if the actual value of μ is 2.1?

5.24 Refer to Exercise 5.23. Compute the power of the test PWR(μ_a) at $\mu_a = 2.1, 2.2, 2.3, 2.4$, and 2.5. Sketch a smooth curve through a plot of PWR(μ_a) versus μ_a.

a. If α is reduced from .05 to .01, what would be the effect on the power curve?

b. If the sample size is reduced from 90 to 50, what would be the effect on the power curve?

5.5 **Choosing the Sample Size for Testing μ**

Med. **5.25** A national agency sets recommended daily dietary allowances for many supplements. In particular, the allowance for zinc for males over the age of 50 years is 15 mg/day. The agency would like to determine if the dietary intake of zinc for active males is significantly higher than 15 mg/day. How many males would need to be included in the study if the agency wants to construct an $\alpha = .05$ test with the probability of committing a Type II error to be at most .10 whenever the average zinc content is 15.3 mg/day or higher? Suppose from previous studies they estimate the standard deviation to be approximately 4 mg/day.

Edu. **5.26** To evaluate the success of a 1-year experimental program designed to increase the mathematical achievement of underprivileged high school seniors, a random sample of participants in the program will be selected and their mathematics scores will be compared with the previous year's statewide average of 525 for underprivileged seniors. The researchers want to determine whether the experimental program has increased the mean achievement level over the previous year's statewide average. If $\alpha = .05$, what sample size is needed to have a probability of Type II error of at most .025 if the actual mean is increased to 550? From previous results, $\sigma \approx 80$.

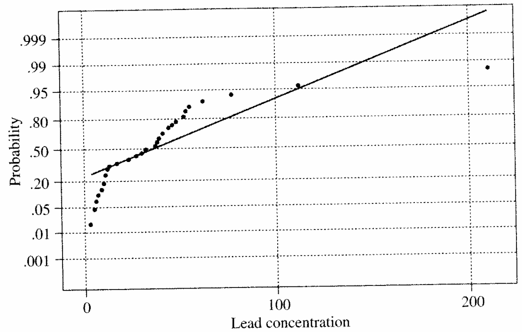
5.27 Refer to Exercise 5.26. Suppose a random sample of 100 students is selected yielding $\bar{y} = 542$ and $s = 76$. Is there sufficient evidence to conclude that the mean mathematics achievement level has been increased? Explain.

Bus. **5.28** The administrator of a nursing home would like to do a time-and-motion study of staff time spent per day performing nonemergency tasks. Prior to the introduction of some efficiency measures, the average person-hours per day spent on these tasks was $\mu = 16$. The administrator wants to test whether the efficiency measures have reduced the value of μ. How many days must be sampled to test the proposed hypothesis if she wants a test having $\alpha = .05$ and the probability of a Type II error of at most .10 when the actual value of μ is 12 hours or less (at least a 25% decrease from prior to the efficiency measures being implemented)? Assume $\sigma = 7.64$.

Env. **5.29** The vulnerability of inshore environments to contamination due to urban and industrial expansion in Mombasa is discussed in the paper "Metals, petroleum hydrocarbons and organochlorines in inshore sediments and waters on Mombasa, Kenya" (*Marine Pollution Bulletin*, 1997, pp. 570–577). A geochemical and oceanographic survey of the inshore waters of Mombasa, Kenya, was undertaken during the period from September 1995 to January 1996. In the survey, suspended particulate matter and sediment were collected from 48 stations within Mombasa's estuarine creeks. The concentrations of major oxides and 13 trace elements were determined for a varying number of cores at each of the stations. In particular, the lead concentrations in suspended particulate matter (mg kg^{-1} dry weight) were determined at 37 stations. The researchers were interested in determining whether the average lead concentration was greater than 30 mg kg^{-1} dry weight. The data are given in the following table along with summary statistics and a normal probability plot.

Lead concentrations (mg kg^{-1} dry weight) from 37 stations in Kenya

48	53	44	55	52	39	62	38	23	27
41	37	41	46	32	17	32	41	23	12
3	13	10	11	5	30	11	9	7	11
77	210	38	112	52	10	6			



a. Is there sufficient evidence ($\alpha = .05$) in the data that the mean lead concentration exceeds 30 mg kg^{-1} dry weight?

b. What is the probability of a Type II error if the actual mean concentration is 50?

c. Do the data appear to have a normal distribution?

d. Based on your answer in (c), is the sample size large enough for the test procedures to be valid? Explain.

5.6 The Level of Significance of a Statistical Test

Eng.

5.30 An engineer in charge of a production process that produces a stain for outdoor decks has designed a study to test the research hypotheses that an additive to the stain will produce an increase in the ability of the stain to resist water absorption. The mean absorption rate of the stain without the additive is $\mu = 40$ units. The engineer places the stain with the additive on $n = 50$ pieces of decking material and records $\bar{y} = 36.5$ and $s = 13.1$. Determine the level of significance for testing H_a: $\mu < 40$. Is there significant evidence in the data to support the contention that the additive has decreased the mean absorption rate of the stain using an $\alpha = .05$ test?

5.31 Refer to Exercise 5.30. If the engineer used $\alpha = .025$ in place of $\alpha = .05$, would the conclusion about the research hypothesis change? Explain how the same data can reach a different conclusion about the research hypothesis.

Env.

5.32 A concern to public health officials is whether a concentration of lead in the paint of older homes may have an effect on the muscular development of young children. In order to evaluate this phenomenon, a researcher exposed 90 newly born mice to paint containing a specified amount of lead. The number of Type 2 fibers in the skeletal muscle was determined 6 weeks after exposure. The mean number of Type 2 fibers in the skeletal muscles of normal mice of this age is 21.7. The $n = 90$ mice yielded $\bar{y} = 18.8$, $s = 15.3$. Is there significant evidence in the data to support the hypothesis that the mean number of Type 2 fibers is different from 21.7 using an $\alpha = .05$ test?

5.33 Refer to Exercise 5.32. In fact, the researcher was more concerned about determining if the lead in the paint reduced the mean number of Type 2 fibers in skeletal muscles. Does the change in the research hypothesis alter your conclusion about the effect of lead in paint on the mean number of Type 2 fibers in skeletal muscles?

Med.

5.34 A tobacco company advertises that the average nicotine content of its cigarettes is at most 14 milligrams. A consumer protection agency wants to determine whether the average nicotine content is in fact greater than 14. A random sample of 300 cigarettes of the company's brand yield an average nicotine content of 14.6 and a standard deviation of 3.8 milligrams. Determine the level of significance of the statistical test of the agency's claim that μ is greater than 14. If $\alpha = .01$, is there significant evidence that the agency's claim has been supported by the data?

Psy.

5.35 A psychological experiment was conducted to investigate the length of time (time delay) between the administration of a stimulus and the observation of a specified reaction. A random sample of 36 persons was subjected to the stimulus and the time delay was recorded. The sample mean and standard deviation were 2.2 and .57 seconds, respectively. Is there significant evidence that the mean time delay for the hypothetical population of all persons who may be subjected to the stimulus differs from 1.6 seconds? Use $\alpha = .05$. What is the level of significance of the test?

5.7 Inferences about μ for a Normal Population, σ Unknown

5.36 Set up the rejection region based on the t statistic for the following research hypotheses:

a. H_a: $\mu > \mu_0$, use $n = 12$, $\alpha = .05$

b. H_a: $\mu < \mu_0$, use $n = 23$, $\alpha = .025$

c. H_a: $\mu > \mu_0$, use $n = 9$, $\alpha = .001$

d. H_a: $\mu \neq \mu_0$, use $n = 19$, $\alpha = .01$

5.37 A researcher uses a random sample of $n = 17$ items and obtains $\bar{y} = 10.2$, $s = 3.1$. Using an $\alpha = .05$ test, is there significant evidence in the data to support H_a: $\mu > 9$? Place bounds on the level of significance of the test based on the observed data.

Edu.

5.38 The ability to read rapidly and simultaneously maintain a high level of comprehension is often a determining factor in the academic success of many high school students. A school district is considering a supplemental reading program for incoming freshmen. Prior to implementing the program, the school runs a pilot program on a random sample of $n = 20$ students. The students were thoroughly tested to determine reading speed and reading comprehension. Based on a fixed-length standardized test reading passage, the following reading times (in minutes) and increases in comprehension scores (based on a 100-point scale) were recorded.

Student	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	n	$\bar{y}$	s
Reading Time	5	7	15	12	8	7	10	11	9	13	10	6	11	8	10	8	7	6	11	8	20	9.10	2.573
Comprehension	60	76	76	90	81	75	95	98	88	73	90	66	91	83	100	85	76	69	91	78	20	82.05	10.88

a. Place a 95% confidence interval on the mean reading time for all incoming freshmen in the district.

b. Plot the reading time using a normal probability plot or boxplot. Do the data appear to be a random sample from a population having a normal distribution?

c. Provide an interpretation of the interval estimate in part (a).

5.39 Refer to Exercise 5.38. Using the reading comprehension data, is there significant evidence that the reading program would produce for incoming freshmen a mean comprehension score greater than 80, the statewide average for comparable students during the previous year? Provide bounds on the level of significance for your test. Interpret your findings.

5.40 Refer to Exercise 5.38.

a. Does there appear to be a relationship between reading time and reading comprehension of the individual students? Provide a plot of the data to support your conclusion.

b. What are some weak points in this study relative to evaluating the potential of the reading improvement program? How would you redesign the study to overcome these weak points?

Bus.

5.41 A consumer testing agency wants to evaluate the claim made by a manufacturer of discount tires. The manufacturer claims that its tires can be driven at least 35,000 miles before wearing out. To determine the average number of miles that can be obtained from the manufacturer's tires, the agency randomly selects 60 tires from the manufacturer's warehouse and places the tires on 15 cars driven by test drivers on a 2-mile oval track. The number of miles driven (in thousands of miles) until the tires are determined to be worn out is given in the following table.

Car	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	n	$\bar{y}$	s
Miles Driven	25	27	35	42	28	37	40	31	29	33	30	26	31	28	30	15	31.47	5.04