Group Project 2: Spring dynamics

Due date: March 6th

In this project, you will explore the physical behavior of a spring, compare your physical intuition to solutions for spring equations of motion, and fit a spring equation to data.

Learning goals (what you should get out of the project)

- Compare differential equation terms to physical laws of motion (for springs)
- Learn how roots lead to different behavior in constant coefficient equations
- Compare solutions to differential equations with physical behavior (of springs)
- Experiment with fitting ODE solutions to physical data

Topic	Recommended timeline
Explore spring behavior	In class, Feb. 12
Solve auxiliary equation	In class, Feb. 15 and 19
Explain possible behaviors	In class, Feb. 15 and 19
Compare springs and equation	Out of class
Set up spreadsheet	In class, Feb. 26
Test different coefficients	Out of class

Guidelines for team contract

- Groupwork is important and broadly relevant. However, you may be unfamiliar with effective groupwork - formal contracts help prevent issues that can arise in groups.
- You can edit this template for a team contract. You can also make your own.
- Contract is due on blackboard within 24 hours of the project being assigned.
- There is no penalty for skipping a team contract.
- I will only arbitrate groupwork disagreements based on team contracts - otherwise, all groupmates share equal responsibility for whatever happens on the project.

Submission details

- Project is due at the start of class, in person or via blackboard.
- Every group member's name should be on the project.
- **Digital submissions must be ONE document in PDF format**.
  Multiple or non-pdf files will receive a 0 on the project.
- **It is your responsibility to check that digital submissions are correct.**
  If the file is corrupt or for a different class, you will receive a 0 on the project.

Tasks to complete

- Explore spring models (see in-class activity on Feb. 12)
  - Describe expected behavior of a spring from your physical experience (1s)
  - Propose some functions *x(t)* that match this behavior
  - Sketch your solutions (or use desmos) to show what you think *x(t)* looks like
  - Briefly explain the terms in the ODE for a spring (1s)
  - Repeat these steps for undamped *and* damped springs

- Solve the equation for a damped spring
  - The ODE for a 1 gram mass attached to a spring with coefficient 20 dyne/cm and unknown damping coefficient β is given by: $x'' + \beta x' + 20x = 0$
  - Solve this ODE up to when you find the roots for your auxiliary equation
  - Check how β can lead to different types of roots (i.e. imaginary, repeated)
  - Describe the **types of behavior** that are possible for different values of β (3s)

- Compare your answers so far
  - Look back at your work on the "Introduction to Springs" handout.
  - Compare your early guesses for *x(t)* to the possible behavior from your roots (2s)

- Finding a solution
  - The general solution for the behavior is: $x = e^{at}(c_1 cos(bt) + c_2 sin(bt))$
  - Write the formula for *a* and *b* from your roots $m = a + bi$.
  - If $x(0) = 2,\ x'(0) = 0$, solve for what $c_1,\ c_2$ should be.
  - Type the formulas for $c_1,\ c_2,\ a,\ b$ into this spreadsheet. Use J8 for unknown β

- Comparing to data
  - Someone released this spring from rest 2cm below the equilibrium position and recorded this data for one second. (Initial conditions $x(0) = 2,\ x'(0) = 0$)

t sec.	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$x(t)$cm.	2.16	1.77	1.40	1.08	0.71	0.39	0.18	0.03	-0.06	-0.10

Data from Brian Winkel, for a SIMIODE project on differential equations

  - This data is for an underdamped spring.
  - What kind of root leads to that behavior?
  - Calculate the range of possible β that will give you that type of root.
  - Experiment with different values of β to fit the data (only modify cell J8).
  - Explain some values you tried and the results of those attempts (2s).
  - Report your best-fit β value and include an image that shows the fit to data.

Rubric (how to earn points)

Points	Topic	Components
5	Introduction / Connection	(1) Discuss behavior a spring should have (1) Discuss terms in the spring ODE (2) Propose/show a range of possible solutions (1) Compare results from spring solution to initial guess
5	Solving spring equation	(1) Solve damped spring equation (2) Find ranges for beta to get different kinds of roots (2) Connect different type of roots to different behaviors
5	Solving damped IVP	(1) Identify a from the spring equation solution (1) Identify b from the spring equation solution (3) Identify c1 and c2 for given initial conditions *answers need to be in terms of unknown beta
5	Numerical fit	(1) Identify correct type of root for the data (1) Identify range of beta to use or the data (2) Set up spreadsheet with answers for a,b,c1,c2 (1) Find best fitting beta for the data