

MODELLING IMPACT OF DIVERSE VEGETATION ON CROP-POLLINATOR
INTERACTIONS

by

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ABSTRACT

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In agricultural landscapes, insects improve crop yield by providing pollination services. The magnitude of pollination services is directly influenced by the abundance of insect populations in the area. Local insect populations are affected by the management of agricultural landscapes and available natural resources. In this work, we explore the potential for incorporating natural vegetation near cotton crops to increase local bee populations and we expect to observe improved cotton production. We begin by creating a compartmental ordinary differential equation model for pollinator-resource interactions across the two types of vegetation to match with data currently being collected in the field. To analyze our model, we calculate points of equilibrium and stability with respect to model parameters and investigate the impact of specific parameters on cotton growth.

To match current research in the field, we incorporate a third land patch between the cotton and natural vegetation patches. The third land patch contains a variety of planted flowers, with the different varieties organized in rows. We study

the specialist and generalist pollinator competition across the land patches and its impact on cotton yields. We model the crop-pollinator interactions over a single cotton season.

To involve students in mathematical modelling, we create a complementary outreach activity for high school students. In the activity students engage with a simulation of pollinator-resource interactions to develop hypotheses and draw conclusions about their impact. The simulation's design incorporates the assumptions of the first mathematical model with the cotton and natural vegetation land patches, in the context of a tomato garden. The outreach activity provides an opportunity for students to better understand mathematical modelling and its benefits beyond the classroom.

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CHAPTER 1

INTRODUCTION

1.1 Biological System

We explore crop-pollinator interactions and the potential to increase local pollinator populations by incorporating natural vegetation and flowers near crop land. Our interest in crop-pollinator interactions is for different bee species and cotton. Pollination is necessary in the process of cotton production. Pollination of cotton may occur naturally, by self-pollination, or from wild pollinators. Although cotton is a self-pollinator, the quality and number of seeds can improve with increased pollination by insects [1].

We have access to land in Terry County, Texas, which is just outside of Lubbock, Texas, to plant cotton near Conservation Reserve Program (CRP) land that has been established for twelve years. The CRP is a government funded program overseen by the Farm Service Agency (FSA). The program incentivizes farmers to use agricultural land for native vegetation, which helps improve the health of the land and environment. The FSA cites the goal of the CRP as “re-establish[ing] valuable land cover to help improve water quality, prevent soil erosion, and reduce loss of wildlife habitat [2].” For the purposes of our research, we believe the CRP and its close proximity to the cotton crop will increase bee populations and, therefore, the pollination services that the bees provide to the cotton.



Figure 1.1. Cotton Patch and Flower Strip.



Figure 1.2. CRP Patch.

The plot of land with the cotton, flower strip, and CRP is continuous, with no obstacles or separation between the three patches. The cotton patch is approximately four acres, with the CRP measuring approximately thirty acres in size. The flower patch is a narrow strip between the cotton and CRP patches.

In our work, we first model and analyze the system by considering only the CRP and cotton patch [Chapter 2, Section 2.3]. We then adapt our initial model to reflect the three patch system (CRP, flower strip, and cotton) [Chapter 2, Section 2.4]. In addition to the three patch model, we consider the pollination services of

generalist and specialist bee species. We model the indirect competition of generalist and specialist bees and the impact of their pollination on cotton growth [Chapter 2, Section 2.5]. The CRP and cotton patch model is the basis of our outreach activity for high school students to promote real-world applications of mathematical modelling and the impact of plant varieties on pollinators [Chapter 3].

CHAPTER 2

POLLINATOR-RESOURCE MODELLING

2.1 Introduction

The objective of our research is to study the potential for incorporating natural vegetation near cotton crops to increase local bee populations and we expect to observe an increase in cotton production. Without the services that bees provide, crop yields would be devastated. Thus, it is important to study ways in which we can promote the presence of bees in agricultural landscapes. We create mathematical models to analyze these impacts, to match the expected pollination services of bees in cotton fields near Lubbock, Texas. Adjacent to the cotton, there is land that the farmers designate to the Conservation Reserve Program (CRP) to allow the land to recover from agricultural production. The CRP supports natural regrowth of plant species that improve the soil and overall land quality [2].

To model the biological system, we first consider a two land patch scenario. The two land patches we consider are the CRP and cotton, located side-by-side. In the second iteration of our model, we consider a three land patch scenario. The third land patch is a narrow strip of flowers where researchers plant and organize different flower types in rows. The flower patch separates the CRP and cotton. With the addition of the flower patch, we recognize the impact of specialist and generalist bee populations on cotton growth.

In this chapter, we model the bee populations in each land patch and the growth of cotton as a result of bee pollination. Additionally, we model specialist and generalist bee competition and impact on cotton production.

2.2 General Assumptions

We consider the pollinator-resource interactions for all models with the following assumptions:

1. Bees provide pollination services to all resources across all plots
2. All resources we consider have the ability to self-pollinate
3. G describes the growth benefit that bees provide to all resources
4. All plant populations experience death due to natural causes (e.g. weather, soil quality)
5. All plots of land have carrying capacities
6. Bees move freely between all resources

2.3 Spatial Two Patch Model

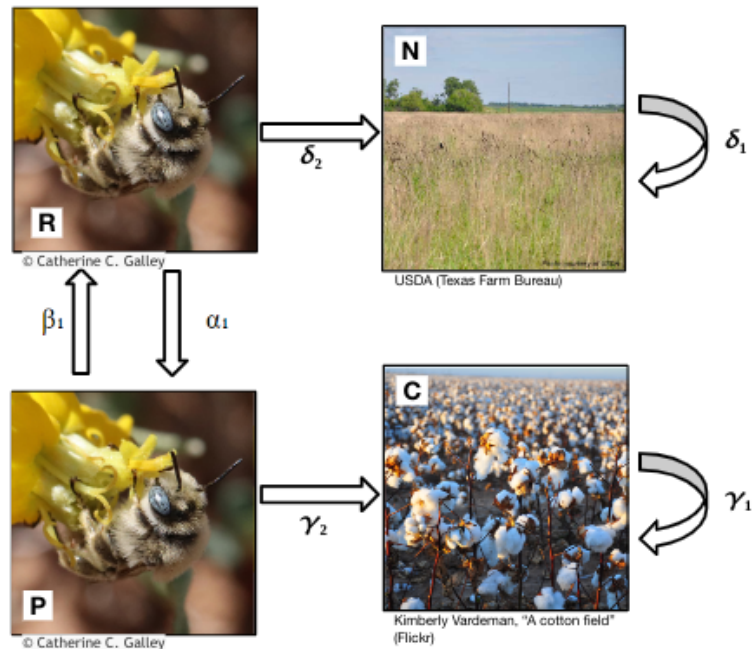


Figure 2.1. Two Patch Scenario Diagram.

We have two resources, the CRP and cotton. Figure 2.1 depicts the interactions and assumptions of the model.

2.3.1 Mathematical Model

We consider the pollinator-resource interactions across the two types of vegetation as a continuous differential equation model.

$$\begin{aligned} \frac{dP}{dt} &= \alpha_1 R - \beta_1 P && \text{(bees in C)} \\ \frac{dR}{dt} &= \beta_1 P - \alpha_1 R && \text{(bees in N)} \\ \frac{dC}{dt} &= (\gamma_1 + \gamma_2 P)C(1 - \frac{C}{k}) - \mu C && \text{(cotton)} \\ \frac{dN}{dt} &= (\delta_1 + \delta_2 R)N(1 - \frac{N}{m}) - \zeta N && \text{(reserve)} \\ \frac{dG}{dt} &= \gamma_2 PC + \delta_2 RN && \text{(bees growth)} \end{aligned}$$

2.3.2 Results

2.3.2.1 Analysis

To analyze the two patch scenario, we calculate points of equilibrium and apply the trace-determinant test to determine stability. We consider two states at a time to compute equilibria (P and R , C and P , R and N). We can decouple the states for equilibrium analysis since the variables do not interact with each other outside of the two states we choose to consider. To implement the trace-determinant test, we calculate the Jacobian for each pair of states we consider. The equilibria and Jacobian results are shown in Table 2.1.

In our analysis of the relationship of bee activity in the cotton (P) and CRP (R) plots, the trace-determinant test is inconclusive, as the determinant of the Jacobian is zero. For further analysis, we look at the phase plane in the next section [Section

Table 2.1. Two Patch Scenario Equilibria and Jacobian

States	Points of Equilibria	Jacobian
P, R	$(0, 0)$ $(P, \frac{\beta_1}{\alpha_1}P)$	$J_{PR} = \begin{bmatrix} -\beta_1 & \alpha_1 \\ \beta_1 & -\alpha_1 \end{bmatrix}$
P, C	$(0, 0)$ $(P, k(1 - \frac{\mu}{\gamma_1 + \gamma_2 P}))$	$J_{PC} = \begin{bmatrix} -\beta_1 & 0 \\ \gamma_2 C(1 - \frac{C}{k}) & \gamma_1 + \gamma_2 P - \frac{2\gamma_1 C + 2\gamma_2 PC}{k} - \mu \end{bmatrix}$
R, N	$(0, 0)$ $(R, m(1 - \frac{\zeta}{\delta_1 + \delta_2 R}))$	$J_{RN} = \begin{bmatrix} -\alpha_1 & 0 \\ \delta_2 N(1 - \frac{N}{m}) & \delta_1 + \delta_2 R - \frac{2\delta_1 N + 2\delta_2 RN}{m} - \zeta \end{bmatrix}$

2.3.2.2]. The analysis of the two other systems we consider yield parameter criteria for system stability.

2.3.2.2 Computation

We use MATLAB to create a phase portrait [Figure 2.2] for the two patch scenario (P and R). In the phase portrait, we observe that the system is attracted to a line of stable fixed points with the ratio $\frac{\beta_1}{\alpha_1}$.

We use numerical ODE solutions via MATLAB to obtain Figure 2.3. The diagram shows the influence of bee pollination on average cotton growth across a single season. We see that initially bee pollination has a large impact on the average cotton growth, but its influence becomes less impactful as cotton approaches the carrying capacity.

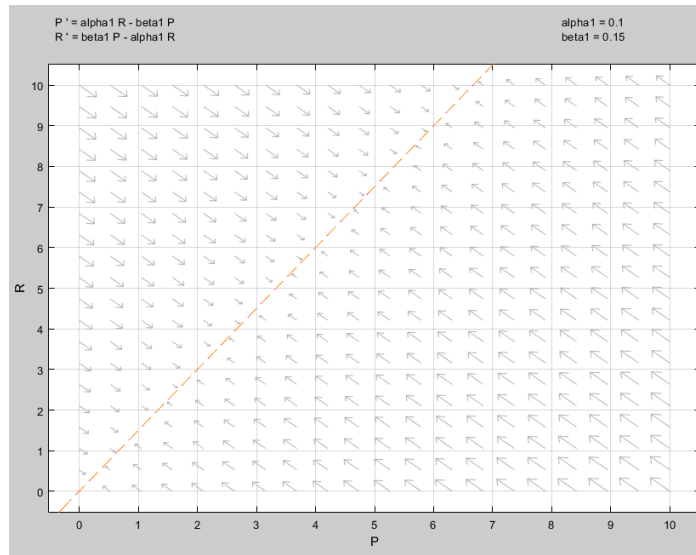


Figure 2.2. Phase Portrait for P and R .

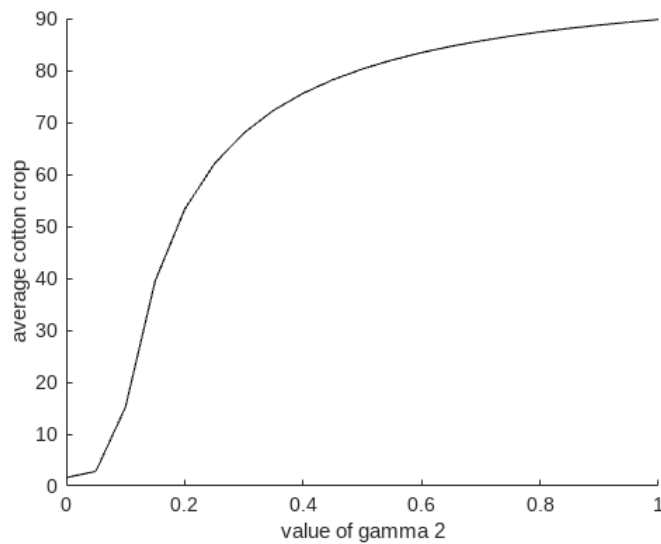


Figure 2.3. Impact of Bee Pollination on Avg Cotton Growth During a Season.

The results we observe from the phase portrait [Figure 2.2] tell us that the bee populations in the CRP patch and the cotton patch have the same ratio, but varying abundances. These abundances depend on the local bee populations. Figure

2.3 informs us that bee pollination services do have a positive impact on the average cotton growth during a single season.

2.4 Spatial Three Patch Model

We have three resources, the cotton, flowers, and CRP. The flower strip is between the CRP and cotton land patches. Figure 2.4 depicts the interactions and assumptions of the model.

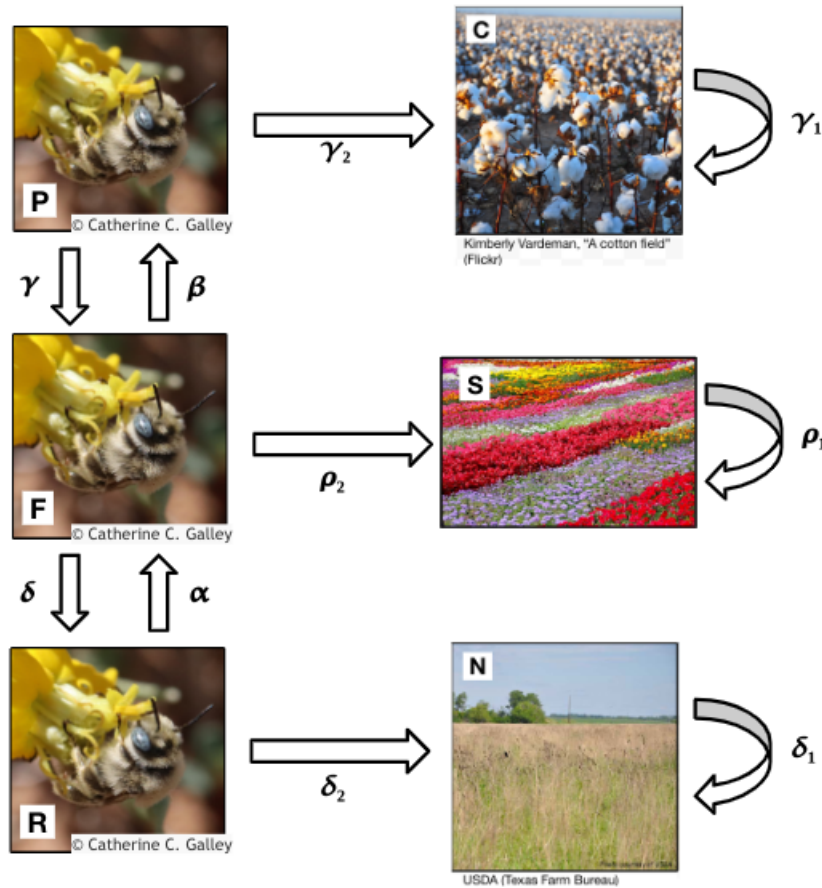


Figure 2.4. Three Patch Scenario Diagram.

2.4.1 Mathematical Model

We consider the three patch scenario and incorporate the flower strip in our previous model. We model the pollinator-resource interactions as a continuous differential equation model.

$$\begin{aligned}
 \frac{dP}{dt} &= \beta F - \gamma P && \text{(bees in C)} \\
 \frac{dF}{dt} &= \alpha R + \gamma P - (\beta + \delta) F && \text{(bees in S)} \\
 \frac{dR}{dt} &= \delta F - \alpha R && \text{(bees in N)} \\
 \frac{dC}{dt} &= (\gamma_1 + \gamma_2 P) C \left(1 - \frac{C}{k}\right) - \mu C && \text{(cotton)} \\
 \frac{dS}{dt} &= (\rho_1 + \rho_2 F) S \left(1 - \frac{S}{l}\right) - \chi S && \text{(flower strip)} \\
 \frac{dN}{dt} &= (\delta_1 + \delta_2 R) N \left(1 - \frac{N}{m}\right) - \zeta N && \text{(reserve)} \\
 \frac{dG}{dt} &= \gamma_2 PC + \rho_2 FS + \delta_2 RN && \text{(bees growth)}
 \end{aligned}$$

2.4.2 Results

2.4.2.1 Analysis

To analyze the three patch scenario, we focus on the bee populations for each of the vegetation land patches. We calculate points of equilibrium for the three states R , F , and P . To determine stability of the system, we calculate the Jacobian and implement the Ruth-Hurwitz Criterion. The equilibria and Jacobian results are shown in Table 2.2.

To implement the Ruth Hurwitz Criterion, we find the characteristic polynomial of the Jacobian in Table 2.2. The characteristic polynomial has no constant term, which means it has an eigenvalue of 0. The non-negative eigenvalue indicates that the system including P , F , and R is not stable and the Ruth Hurwitz Criterion is not necessary. We conclude that this agrees with our analysis of the two patch spatial model. In the two patch spatial model, we find a line of stable points the

Table 2.2. Three Patch Scenario Equilibria and Jacobian

States	P, F, R
Points of Equilibria	$(0, 0, 0)$ $(\frac{\beta\alpha}{\gamma\delta}R, \frac{\alpha}{\delta}R, R)$
Jacobian	$J_{PFR} = \begin{bmatrix} -\gamma & \beta & 0 \\ \gamma & -\beta - \delta & \alpha \\ 0 & \delta & -\alpha \end{bmatrix}$ <p>Characteristic Polynomial: $p(\lambda) = \lambda^3 + (\alpha + \beta + \gamma + \delta)\lambda^2 + (\alpha\beta + \alpha\gamma + \gamma\delta)\lambda$</p>

system approaches and we may have a similar situation for finding stability in the three patch spatial model.

2.4.2.2 Computation

We explore the attractiveness of cotton crops to bees. To explore attractiveness of cotton crops, we observe the parameters $r_1 = \frac{\delta}{\alpha}$ (bees entering CRP patch) and $r_2 = \frac{\beta}{\gamma}$ (bees entering cotton patch). In MATLAB, we calculate the percent of bees in the cotton field using the equilibrium point $(\frac{\beta\alpha}{\gamma\delta}R, \frac{\alpha}{\delta}R, R)$ [Table 2.2]. We generate the following equation to calculate the percentage:

$$\text{Percent of Bees in Cotton} = \frac{P^*}{R^* + F^* + P^*} = \frac{r_2 \cdot (r_1)^{-1}}{1 + (r_1)^{-1} + r_2 (r_1)^{-1}}$$

We create a heat map to compare the parameters r_1 and r_2 and analyze the cotton's attractiveness to bees. We observe from the heat map in Figure 2.5 that the greater the attractiveness of CRP (r_1), the lower the percentage of bees in the cotton field. Alternatively, the greater the attractiveness of cotton (r_2), the higher the percentage of bees in the cotton field. Note that when the attractiveness of the two patches are equal ($r_1 = r_2$), we observe the percent of bees in the cotton field is near 50%.

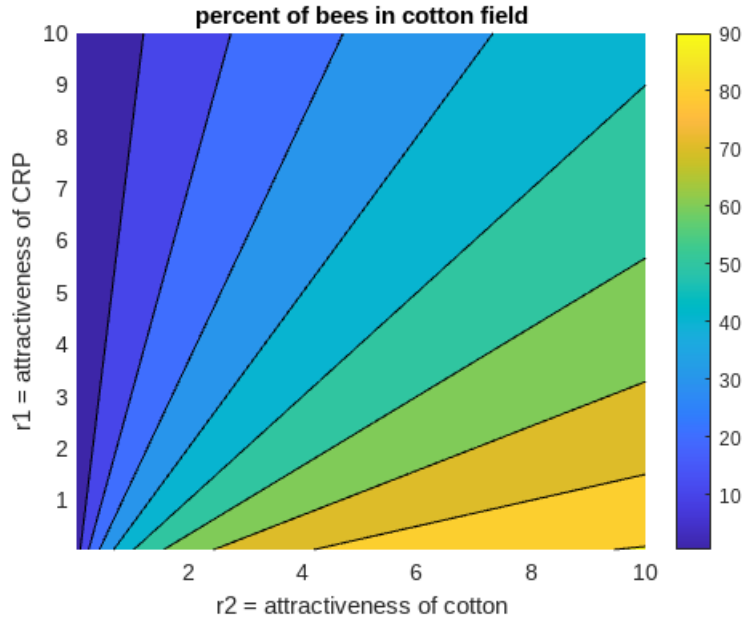


Figure 2.5. Heat Map: Attractiveness of CRP vs. Cotton.

The results in Figure 2.5 demonstrate the percentage of bees in the cotton patch by comparing the attractiveness of the CRP and cotton to bees. These results are important to explore, since the amount of bees in the cotton patch directly affects the pollination services the cotton receives.

2.5 Competition Model

2.5.1 Assumptions

We consider the general assumptions we outline in Section 2.2. To study the competition between bee species in the spatial model, we consider generalist and specialist bees. We add the two assumptions to consider competition between the two bees species:

1. Generalist bees provide pollination services to all plant populations
2. Specialist bees provide pollination services only to flowers

2.5.2 Mathematical Model

First, we consider the indirect competition between generalist (r) and specialist (s) bees during a single season. We want to think about a single season, so we keep both bee populations constant. We are not modelling bee populations because they are constant. We model the dynamics with a compartmental, continuous differential equations model. In the model, we track the plant growth that is a result of generalist and specialist bee pollination services.

$$\begin{aligned} \frac{dC}{dt} &= C[(\gamma_1 + \gamma_2 r)(1 - \frac{C}{k}) - \mu C] && \text{(cotton)} \\ \frac{dN}{dt} &= N[(\delta_1 + \delta_2 r + \delta_3 s)(1 - \frac{N}{m}) - \zeta N] && \text{(natural vegetation and flowers)} \\ \frac{dG_r}{dt} &= \delta_2 r N + \gamma_2 r C && \text{(generalist bees growth)} \\ \frac{dG_s}{dt} &= \delta_3 s N && \text{(specialist bees growth)} \end{aligned}$$

Next, we consider the indirect competition between generalist and specialist bees over an extended period of time, where bee populations are not constant, but are time-dependent functions. We explicitly model the growth and decline of bees to highlight the indirect competition. We model competition with a compartmental, continuous differential equations model.

$$\begin{aligned} \frac{dR}{dt} &= \delta_2 R N + \gamma_2 R C - \eta_1 R && \text{(generalist bees)} \\ \frac{dS}{dt} &= \delta_3 S N - \eta_2 S && \text{(specialist bees)} \\ \frac{dC}{dt} &= C[(\gamma_1 + \gamma_2 R)(1 - \frac{C}{k}) - \mu] && \text{(cotton)} \\ \frac{dN}{dt} &= N[(\delta_1 + \delta_2 R + \delta_3 S)(1 - \frac{N}{m}) - \zeta] && \text{(CRP and flower strip)} \end{aligned}$$

2.5.3 Results

2.5.3.1 Single Season Competition Analysis

To analyze the competition between generalist and specialist bees over a single season, we solve the ODE for $C(t)$ and $N(t)$ using substitution and an integrating factor. As a result we find:

$$C(t) = \frac{1}{\frac{\alpha}{k(\alpha-\mu)} + be^{-(\alpha-\mu)t}} \quad \text{and} \quad N(t) = \frac{1}{\frac{\beta}{m(\beta-\zeta)} + ae^{-(\beta-\zeta)t}}$$

where $\alpha = \gamma_1 + \gamma_2 r$, $\beta = \delta_1 + \delta_2 r + \delta_3 s$, and a, b are constants as a result of integration. Next, we calculate the values of a and b using initial conditions for C and N , assuming their starting abundance is 10% of capacity:

$$C(0) = 0.1\left(\frac{k(\alpha-\mu)}{\alpha}\right) \quad \text{and} \quad N(0) = 0.1\left(\frac{m(\beta-\zeta)}{\beta}\right)$$

We find $a = \frac{0.9\alpha}{0.1k(\alpha-\mu)}$ and $b = \frac{0.9\beta}{0.1m(\beta-\zeta)}$. To compare plant growth as a result of generalist bees versus specialist bees, we input the functions $C(t)$ and $N(t)$ in $\frac{dG_s}{dt}$ and $\frac{dG_r}{dt}$ and solve for G_r and G_s .

$$G_S = \frac{\delta_3 s m}{\beta} \ln |\beta(e^{\beta-\zeta}t + am)|$$

$$G_R = \frac{\delta_2 r m}{\beta} \ln |\beta(e^{\beta-\zeta}t + am)| + \frac{\gamma_2 r k}{\alpha} \ln |\alpha(e^{\alpha-\mu}t + bk)|$$

We consider the parameter values in Table 2.3, to obtain the functions:

$$G_S = \frac{\delta_3 s}{\delta_2 r + \delta_3 s} \ln |(\delta_2 r + \delta_3 s)(e^{(\delta_2 r + \delta_3 s)t} + 9)|$$

$$G_R = \frac{\delta_2 r}{\beta} \ln |(\delta_2 r + \delta_3 s)(e^{(\delta_2 r + \delta_3 s)t} + 9)| + \ln |\gamma_2 r(e^{\gamma_2 r}t + 9)|$$

Table 2.3. Short-Term Competition Model Parameter Values

Parameter Values	Explanation
$m = k = 1$	Each patch is at 100% capacity
$\gamma_1 = \delta_1 = 0$	Natural vegetation and cotton need pollinators
$\mu = \zeta = 0$	No death of plant populations, we focus on growth

2.5.3.2 Single Season Competition Computation

In MATLAB, we explore the difference in growth from generalist and specialist bees ($G_R - G_S$). As a result we observe the heat map in Figure 2.6.

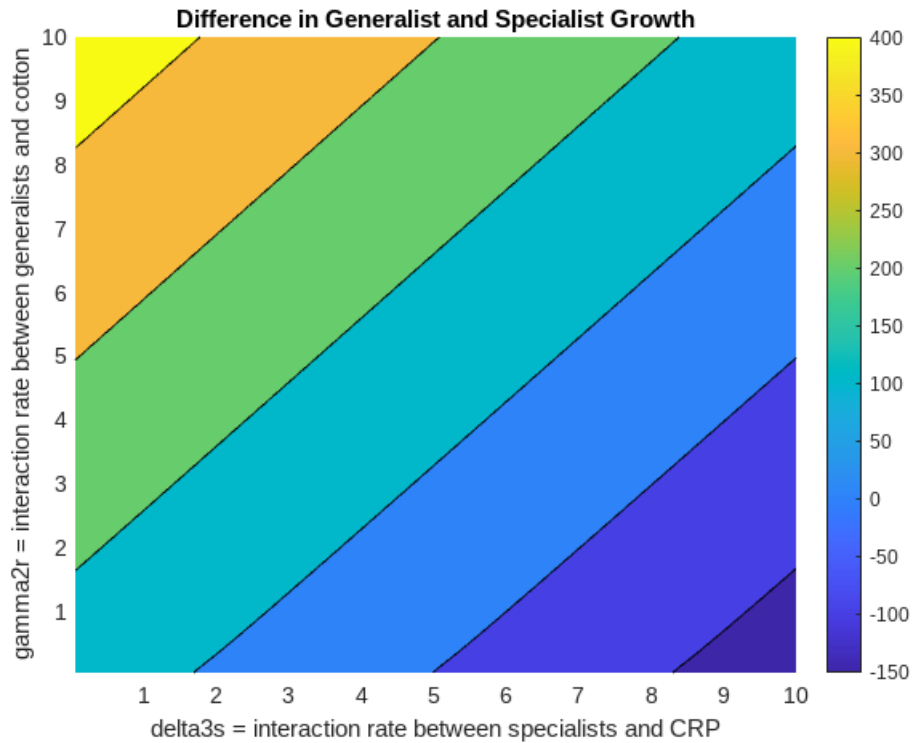


Figure 2.6. Heat Map: Difference in Generalist and Specialist Growth.

We observe that as the interaction rate between specialists and CRP increases, the growth in the specialist population increases. We see this in the decreasing behavior of the difference in generalist and specialist growth as δ_3s increases. We consider the effect of γ_2r and note that as the interaction rate between generalists and cotton increases, the growth in the generalist population increases. This result is reasonable, in that the generalist bees experience growth from both cotton and CRP interactions, whereas specialists benefit only from the CRP. For specialist bee

growth to be dominant, the interaction rate $\delta_3 s$ must be high ($\delta_3 s > 5$), with a low $\gamma_2 r$ interaction rate ($\gamma_2 r < 5$).

Specialist bee populations are important from a conservationist standpoint. Whereas generalist bee populations are of interest to farmers, as they are important for cotton production. The results we observe in Figure 2.6 show the benefits of CRP and cotton to both bee populations to inform conservationist and farmer interests.

2.5.3.3 Long Term Competition Analysis

To analyze the competition between generalist and specialist bees over an extended period of time, we first calculate points of equilibrium. We find the following equilibria for our system (R, S, C, N) :

1. $(0, 0, 0, 0)$
2. $(\theta, \Delta, \frac{1}{\gamma_2}[\eta_1 - \frac{\delta_2 \eta_2}{\delta_3}], \frac{\eta_2}{\delta_3})$
3. $(0, 0, k[1 - \frac{\mu}{\gamma_1}], m[1 - \frac{\zeta}{\delta_1}])$
4. $(0, \frac{1}{\delta_3}[\frac{\zeta(\frac{\eta_2}{\delta_3})}{\frac{\eta_2}{\delta_3}(1 - \frac{\eta_2}{\delta_3 m})} - \delta_1], 0, \frac{\eta_2}{\delta_3})$
5. $(0, \frac{1}{\delta_3}[\frac{\zeta(\frac{\eta_2}{\delta_3})}{\frac{\eta_2}{\delta_3}(1 - \frac{\eta_2}{\delta_3 m})} - \delta_1], \frac{k[\gamma_1 - \mu]}{\gamma_1}, \frac{\eta_2}{\delta_3})$
6. $(0, 0, \frac{k[\gamma_1 - \mu]}{\gamma_1}, 0)$
7. $(\frac{1}{\gamma_2}[\frac{\mu(\frac{\eta_1}{\gamma_2})}{\frac{\eta_1}{\gamma_2}(1 - \frac{\eta_1}{\gamma_2 k})} - \gamma_1], 0, \frac{\eta_1}{\gamma_2}, 0)$
8. $(0, 0, 0, \frac{m}{\delta_1}[\delta_1 - \zeta])$
9. $(\frac{\zeta}{\delta_2[1 - \frac{\eta_1}{\delta_2 m}]}, \frac{\delta_1}{\delta_2}, 0, 0, \frac{\eta_1}{\delta_2})$
10. $(R, 0, k[1 - \frac{\mu}{\gamma_1 + \gamma_2 R}], m[1 - \frac{\zeta}{\delta_1 + \delta_2 R}])$

where $\theta = \frac{1}{\gamma_2}[\frac{\mu}{1 - (\frac{1}{\gamma_2 k}[\eta_1 - \frac{\delta_2 \eta_2}{\delta_3}]} - \gamma_1]$ and $\Delta = \frac{1}{\delta_3}[\frac{\zeta}{1 - \frac{\eta_2}{\delta_3 m}} - \delta_1 - \delta_2 \theta]$ in equilibrium point 2. There is another point of equilibrium where generalists, specialists, and CRP persist and cotton desists that requires restrictions on the parameters.

We calculate the Jacobian for the system and evaluate using the equilibria where all states persist. Our next steps are to use the Ruth Hurwitz Criterion to find parameter values that establish stability of the system.

$$J|_{(R,S,C,N)} = \begin{bmatrix} \delta_2 N + \gamma_2 C - \eta_1 & 0 & \gamma_2 R & \delta_2 R \\ 0 & \delta_3 N - \eta_2 & 0 & \delta_3 S \\ \gamma_2 C(1 - \frac{C}{k}) & 0 & [(\gamma_1 + \gamma_2 R)(1 - \frac{C}{k}) - \mu] + \frac{C}{k}(-\gamma_1 - \gamma_2) & 0 \\ \delta_2 N(1 - \frac{N}{m}) & \delta_3 N(1 - \frac{N}{m}) & 0 & [(\delta_1 + \delta_2 R + \delta_3 S)(1 - \frac{N}{m}) - \zeta] - \frac{N}{m}(\delta_1 + \delta_2 R + \delta_3 S) \end{bmatrix}$$

$$J|_{(\theta, \Delta, \frac{1}{\gamma_2}[\eta_1 - \frac{\delta_2 \eta_2}{\delta_3}], \frac{\eta_2}{\delta_3})} = \begin{bmatrix} 0 & 0 & \gamma_2 \theta & \delta_2 \theta \\ 0 & 0 & 0 & \delta_3 \Delta \\ (\eta_1 - \frac{\delta_2 \eta_2}{\delta_3})(1 - \frac{1}{k\gamma_2}[\eta_1 - \frac{\delta_2 \eta_2}{\delta_3}]) & 0 & \frac{1}{k\gamma_2}(\eta_1 - \frac{\delta_2 \eta_2}{\delta_3})(-\gamma_1 - \gamma_2) & 0 \\ \frac{\delta_2 \eta_2}{\delta_3}(1 - \frac{\eta_2}{\delta_3 m}) & \eta_2(1 - \frac{\eta_2}{\delta_3 m}) & 0 & -\frac{\eta_2}{\delta_3 m}(\delta_1 + \delta_2 \theta + \delta_3 \Delta) \end{bmatrix}$$

Further analysis of the long term competition model between generalist and specialist bees will provide more information for our findings in Section 2.5.3.1 and 2.5.3.2.

2.6 Conclusions

In our analysis of the two patch spatial model, we examine the positive impact of bee pollination services on average cotton growth across a season in Figure 2.3. The analysis of the three patch spatial model yields results that show an increase in bee populations in cotton as a consequence of an increase in attractiveness of the crop. Whereas, bee populations in the CRP are more abundant when the natural vegetation is more attractive, as seen in Figure 2.5.

We observe in our analysis of the competition of generalist and specialist bee populations that the CRP is necessary for specialist bee populations in Figure 2.6. Without the CRP and flower strip, there are no resources to attract and support specialist bee populations.

As a result of our analysis of all models, we observe that the incorporation of natural vegetation has a positive impact on bee populations and cotton production. The spatial models support the implications that the pollination services bees provide support cotton growth. Additionally, an increase in the attractiveness of cotton to bees, increases the bee populations in the cotton. The long term competition of generalist and specialist bees supports the need for incorporating natural vegetation and flowers to increase local bee populations, specifically specialist bee populations.

There is more work to be done to understand pollinator-resource interactions. Future directions of the work may include the incorporation of the bee life cycle and studying the impact of its variation throughout the year on crop seasons. It would be beneficial to more accurately represent the impact of plant death rates on bee populations in the land patches. We may explore how we can represent that a higher death rate of the cotton results in an initial decrease in cotton and should therefore result in a decrease in bee interactions with cotton. Lastly, we could explore the impact of how fields are treated (herbicides, fungicides, and insecticides) and the impact these treatments have on bee populations and, therefore, cotton yield.

CHAPTER 3

OUTREACH ACTIVITY

3.1 Introduction

To emphasize the applications of mathematics, we develop an activity for high school students that centers around a research question with which students can relate and draw connections to their individual lives. We invite students to consider the question, “How does what you plant affect pollinators?” The question piques student interest and focuses student attention to plant-pollinator relationships with a simple question that has large impacts in the agricultural industry. We structure the outreach activity with guided exploration of a variable’s impact on crop yield and pollinator activity with simulations, then challenge students to investigate other variables’ effects. To investigate, students collaborate with peers to create a problem-solving protocol and implement the plan.

We identify learning outcomes that align with the Texas Essential Knowledge and Skills (TEKS) and science communication skills of the activity to encourage high school mathematics classroom participation. The learning outcomes of the activity align with the TEKS for Precalculus and include:

- 111.42(c)(1)(A) apply mathematics to problems arising in everyday life, society, and the workplace;
- 111.42(c)(1)(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying

the solution, and evaluating the problem-solving process and the reasonableness of the solution;

- 111.42(c)(2)(I) determine and analyze the key features of exponential, logarithmic, rational, polynomial, power, trigonometric, inverse trigonometric, and piecewise defined functions, including step functions such as domain, range, symmetry, relative maximum, relative minimum, zeros, asymptotes, and intervals over which the function is increasing and decreasing;
- 111.42(c)(2)(J) analyze and describe end behavior of functions, including exponential, logarithmic, rational, polynomial, and power functions, using infinity notation to communicate this characteristic in mathematical and real-world problems;

In the following chapter we will discuss the Netlogo simulation and the implementation of the activity in the high school mathematics classroom.

3.1.1 Netlogo Simulation

In the activity, students explore the effects of plant varieties on pollinators and the benefits they have on crop yields in a Netlogo simulation. Netlogo is an Agent-Based Modelling (ABM) environment that provides computational descriptions of real-world occurrences. In Netlogo, we define agents and environments to model a desired situation. The environment describes where the specified situation occurs, and the agents are the populace of the environment. To model agent-agent and agent-environment interactions in the simulation, we code commands that execute our assumptions of these interactions. Commands may be stochastic in nature (randomly determined) or logical (if a, then b).

To maximize student interaction with the model, we choose Netlogo for its user-friendly interface. In the Netlogo interface, students use sliders, switches, and buttons to manipulate parameters and run simulations. The agents and environments of the model are represented with icons in the viewing window that move to show agent interactions with the environment. Students make observations from viewing the simulation and collect data from monitor windows and plots in the interface. Netlogo does not need any special software and can be run in any browser, which is more accessible for students in the classroom.

3.2 Implementation

3.2.1 Student Resources

To begin the activity, students consider the question, “How does what you plant affect pollinators?” To make the activity more accessible for students, we reframe the question in terms of a garden. Students focus on the effects of plant varieties on pollinators and use their findings to define ideal conditions that promote the greatest yields in their garden. We provide background information of the bee-plant system of interest. We discuss the assumptions of the model, view a diagram of the bee-plant garden scenario, and introduce the Netlogo simulation.

To manipulate garden parameters in the simulation, students use the sliders in the Netlogo interface. The five parameters that students manipulate are the ratio of tomato plants to flowers (“percentage_tomato_plants”), the initial bee abundance in the garden (“initial-number-bees”), the units of pollen required for a tomato plant to produce a tomato (“tomato_growth”), the rate of bee foraging

Background Information

- Insects improve crop yields by providing pollination services
- Amount of pollination services depends on the abundance of insect populations in the area
- Local insect populations are affected by the management of agricultural landscapes and the natural resources that are available
- We will explore the impact of incorporating flowers in vegetable gardens on bee activity and vegetable yields

The diagram shows a vertical brown rectangle representing a garden. At the top are two purple flowers with a bee flying above them. Below the flowers are two sets of three question marks '???' on a horizontal line. At the bottom are two tomato plants with red tomatoes. To the right of the garden, two bees are shown in a cycle: one at the top and one at the bottom. Arrows indicate a clockwise cycle: from the top bee to the top flowers, from the top flowers to the top bee, from the top bee to the bottom bee, and from the bottom bee to the bottom tomato plants. There are also arrows pointing from the top flowers to the top tomato plants and from the bottom tomato plants to the bottom bee.

Pollinator-Resource Model

Figure 3.1. Activity Slides: Background Information.

(“bee_foraging_rate”), and the number of plants with excess pollen required to attract new bees (“bee_arrival_rate”). To track the abundance of all agents in the garden at any given time, we include monitors that count flowers, tomato plants, flowers with excess pollen, tomato plants with excess pollen, bees, and tomatoes. We include plots in the simulation for observation of the abundance of tomatoes over time (“Plant Yields”) and the frequency of bee visits to flowers and tomato plants (“Bee Visits to Plants”). To view pollen abundance for plants and bees in the model, students use the switch “show-pollen?” The flower pollen count reflects units of pollen available for bees to collect, the tomato plant pollen count reflects units of pollen from bee interactions, and the bee pollen count reflects pollen from tomato plant and flower interactions.

We begin our investigation with group inquiry of the effects of the ratio of tomato plants to flowers on bee activity in the garden and tomato yields. The investigation follows the steps of the scientific method: students develop a hypothesis, experiment using the Netlogo simulation, analyze data, and draw conclusions. We instruct students to manipulate the ratio of tomato plants to flowers in their garden

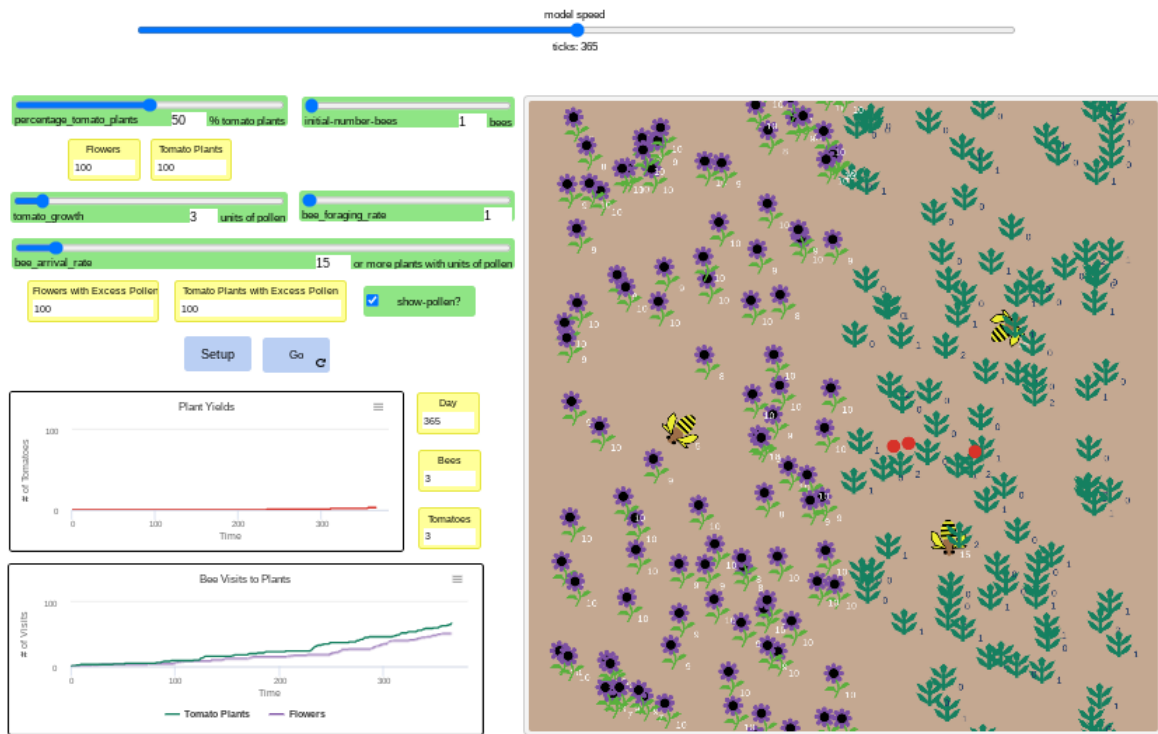


Figure 3.2. Netlogo Interface.

and hold all other parameters constant. To structure student observations and data collection, we include guiding questions, tables, and graphs on the student handout. To aid students in data analysis, we provide questions for student reflection on the experiment.

To conclude the investigation of the effects of the ratio of tomato plants to flowers on bee activity and tomato yields, students share their findings with the class. Similarly, students investigate the effects of the initial bee abundance in their garden on tomato yields. Next, we have students consider all five parameters. To study the effects of all five parameters in the simulation on bee activity and tomato yields, students create a problem-solving plan of action. Students execute the plan in groups and note their findings for each parameter's impact. Additionally, students

- 4) Run the simulation and record observations:
 - a. What do you notice about the “Plant Yields” graph?
 - b. How would you describe tomato growth? (linear, quadratic, exponential, logarithmic, etc.)
 - c. What does the graph tell you about the relationship between time and the # of tomatoes?
 - d. What do you notice about the bee activity in your garden?
- 5) Record tomato yields in the table below, then plot the data on the graph provided.

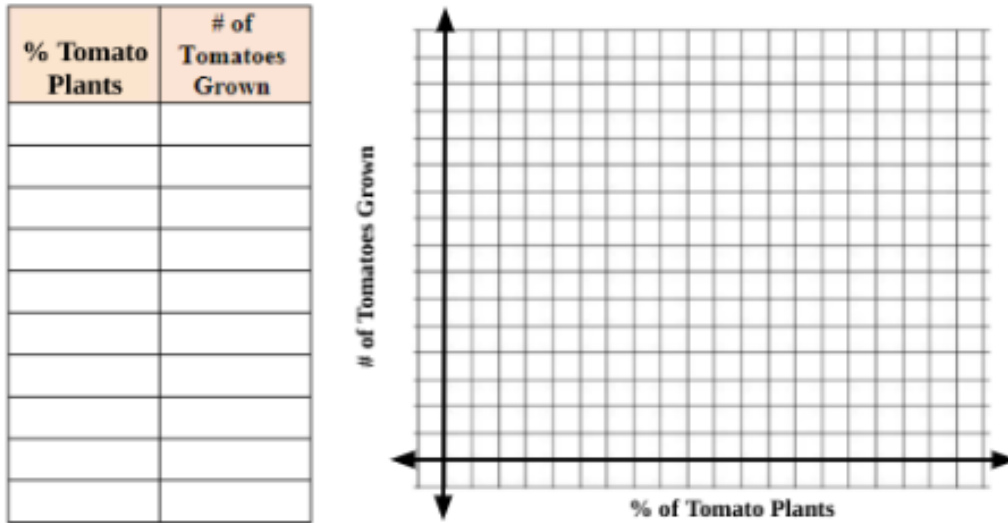


Figure 3.3. Student Handout: Observation Questions.

record their greatest tomato yield and parameter values that achieve the yield.

We conclude the activity with a discussion of pollinator diversity. We choose honeybees for plant pollinators in the simulation to match our mathematical research and common perceptions of pollinators. Contrary to this belief, honeybees do not pollinate tomato plants. We discuss different pollinators and their benefits to plant and crop growth. Additionally, we consider the difference between simulation outcomes and reality. To compare simulations with reality, we have students reflect on their greatest tomato yield and the consistency of simulation outcomes. We ask students, “How realistic is it that you would achieve your tomato yield in an actual

garden? Why or why not?”

3.2.2 Assumptions of Simulation

Our Netlogo simulation adopts the assumptions of the first mathematical model [see section 2.2] through the lens of a vegetable garden. The two land patches are the environment for our ABM and the agents include the bees, plants (tomato plants and flowers), and yield (tomato fruits) populating the garden. The model assumptions for the activity include:

1. Bees pollinate flowers
2. Bees pollinate tomato plants
3. Bees move freely between tomato plants and flowers
4. Tomato plants produce tomatoes as a result of pollination
5. Land can only accommodate so many plants

In Netlogo, we code the simulation to have bees move randomly throughout the garden. While the bees roam about the garden, they pollinate plants in the area. Once tomato plants collect enough pollen from the interactions, they produce a tomato. To attract new bees to the garden, we code the model to count flowers with excess pollen (6 or more units of pollen) that bees can collect. Alternatively, we code the model for bees to leave when flower pollen abundance is low. To account for bee behavior in the field, we also code the bees to leave given the energy they expend (moves bees make), or the total pollen bees collect from both tomato plants and flowers. To incorporate the carrying capacity of a single tomato plant in the code, we only allow for a maximum of six tomatoes to be produced by any one

tomato plant.

3.2.3 Learning Outcomes

Students apply mathematics to the everyday life problem of maximizing the output from a garden. To investigate the problem, students create a problem-solving model to analyze data from observations of the Netlogo simulation to determine and justify a solution for maximizing garden output. Students analyze key features of graphs. The graphs and functions that students analyze in the activity are from Netlogo plots and graphs students create from data and observations. To analyze the plots, students identify key features such as relative minimum, relative maximum, zeros, and intervals over which the function is increasing and decreasing. Students use these features to determine optimal parameter values to achieve maximal garden outputs. Students analyze and describe end behaviors of functions from their graphs. Students can predict future abundance of yields and patterns of simulation behaviors through the analysis of function end behaviors.

We identify science communication skills students use to participate in the activity. Throughout the activity, students investigate natural phenomena with mathematics in a relatable scenario, a vegetable garden. We empower students to communicate findings and collaborate with peers to achieve a common goal. Collaboration and communication are essential skills in the mathematical and scientific community.

3.3 Reflect/Conclude

We implemented the activity in a College Algebra classroom at Monterey High School, Lubbock, Texas on March 5, 2024. Each of the twenty-eight students accessed the Netlogo simulation on their Chromebooks and had a paper copy of the student handout [Appendix C]. To set up the activity, we introduced students to the research question and provided background information that informs the scenario. Next, to engage with the simulation, students downloaded the .nlogo file and uploaded it to Netlogo Web at <https://ccl.northwestern.edu/netlogo/>.

Throughout the activity, student engagement with the simulation was favorable; students enjoyed adjusting the parameter values and observing as bees populated their garden and tomatoes grew. In the Netlogo code, we coded the bees to hatch with a small probability that a hatched bee would be three times the size of a regular size bee in the simulation. We call this oversized bee, the queen bee. Some students encountered the "queen bee" in their garden, and other students tried to replicate the experience. Students made observations and theories as to why flowers were necessary in their gardens to increase/maximize tomato yield. Some theories included:

- "There is more pollen for the bees to collect"
- "Flowers are more desirable"
- "Cross-pollination is needed for tomatoes to grow"

In our whole class discussion, it was difficult for students to reference and show features of their graphs with the class when not all students had the same data points. To strengthen discussion in the future, we plan to have students plot data points on

the board for all to see, reference, and analyze.

After implementation of the activity, we found that time was an issue. All three parts of the activity require at least a day each to have students properly explore all prompts and parameters. To modify the outreach activity to be appropriate for a fifty minute class period, we plan to focus on part one - the exploration of the ratio of tomato plants to flowers and its effect on bee activity and tomato yield. For future class use, we may implement the activity as designed in 3-5 fifty minute class periods.

We designed the activity to be low floor (students of all abilities can participate at the entry level), but we observed that some students finished data collection and analysis before others. To accommodate this and hold student engagement, a future modification will be to add a high ceiling activity for students to explore. In exploring the effect of plant ratio on tomato yield students noted in their data points and graphs that tomato production drops off. We will encourage students to find the exact ratio where tomato production drops and compare with peers to find if the cutoff point is the same for everyone. Once students locate their cutoff point for tomato production, they will express the situation as a limit.

APPENDIX A
NETLOGO SIMULATION CODE

```

globals [
  max-bees ; don't let the bee population grow too large
  max-veggies ; don't let the veggie population grow too large
  max-flowers ; don't let the flower population grow too large
  max-tomatoes
]

; Bees, veggies, and flowers are all breeds of turtles
breed [ bees bee ]
breed [ veggies veggie ]
breed [ flowers flower ]
breed [ tomatoes tomato ]
breed [ beeflowers beflower ]
breed [ beeveggies beeveggie ]

veggies-own [ pollen1 pollen2 fruits ]
flowers-own [ pollen1 pollen2 ]
bees-own [ pollenF pollenV energy ]

to Setup
  clear-all
  ask patches [ set pcolor brown + 2 ]
  create-veggies (percentage_tomato_plants)*(2) ; create the veggies, then initialize
  their variables
  [
    set shape "plant medium"
    set color turquoise - 1
    set size 2
    set pollen1 0
    set pollen2 10
    set fruits 0
    set label-color blue - 2
    setxy random-float ((0)+(15)) random-ycor
  ]

  create-flowers (200)-((2)*(percentage_tomato_plants)) ; create the flowers, then
  initialize their variables
  [
    set shape "flower"
    set color violet
    set size 2
    set pollen1 0
    set pollen2 10
    setxy random-float ((0)-(15)) random-ycor
  ]

  create-bees initial-number-bees ; create the bees, then initialize their variables
  [
    set shape "bee 2"
    set color yellow
    set size 2
    set pollenV 0
    set pollenF 0
  ]

```

```

    set energy 0
    setxy random-xcor random-ycor
]

display-labels
reset-ticks
end

to Go
; stop the model if there are no bees, no veggies, and no flowers
if not any? turtles [ stop ]
; stop the model if there are no bees
if not any? bees [ stop ]

ask bees [
  move

  ; bees visit flowers/veggies, bees gain pollen, flowers/veggies grow
  visit-veggies
  visit-flowers
]
ask one-of bees [
  arrive-bees
  leave-bees
  bees-leave
]
ask veggies [
  ; veggies get pollen from visiting bees, veggies grow
  grow-veggies
  create-pollen
]
ask flowers [
  ; flowers get pollen from visiting bees, flowers grow
  create-pollen
]
tick
display-labels
end

to move
  rt random 10
  lt random 10
  fd bee_foraging_rate
  set energy energy + .01
end

to visit-veggies
  ifelse pollenV >= 1 [
    let plant one-of veggies-here
    if plant != nobody and any? veggies [
      ask plant [ ifelse pollen2 >= 1 [
        set pollen2 pollen2 - 1
        set pollen1 pollen1 + 1

```



```

        ask one-of bees-here [ set pollenV pollenV + 1 ]
      ] [ set pollen1 pollen1 + 1 ]
    ]
    set pollenV pollenV - .1
    hatch-beeveggies 1 [
      rt random-float 360 fd 1
      set shape "circle"
      set color lime
      set size 0
    ]
  ]] [
let plant one-of veggies-here
if plant != nobody and any? veggies [
  ask plant [ ifelse pollen2 >= 1 [
    set pollen2 pollen2 - 1
    ask one-of bees-here [ set pollenV pollenV + 1 ]
    hatch-beeveggies 1 [
      rt random-float 360 fd 1
      set shape "circle"
      set color lime
      set size 0
    ]
  ] [ set pollen1 pollen1 + 0 ]
]
]
]
end

```

```

to visit-flowers
ifelse pollenF >= 1 [
let plant one-of flowers-here
if plant != nobody and any? flowers [
  ask plant [ ifelse pollen2 >= 1 [
    set pollen2 pollen2 - 1
    set pollen1 pollen1 + 1
    ask one-of bees-here [ set pollenF pollenF + 1 ]
  ] [ set pollen1 pollen1 + 1 ]
]
set pollenF pollenF - .1
hatch-beeflowers 1 [
rt random-float 360 fd 1
set shape "circle"
set color violet
set size 0
]
]] [
let plant one-of flowers-here
if plant != nobody and any? flowers [
  ask plant [ ifelse pollen2 >= 1 [
    set pollen2 pollen2 - 1
    ask one-of bees-here [ set pollenF pollenF + 1 ]
    hatch-beeflowers 1 [
      rt random-float 360 fd 1
    ]
  ]
]
]
end

```

```

        set shape "circle"
        set color violet
        set size 0
    ]
    ] [ set pollen1 pollen1 + 0 ]
]
]
end

to arrive-bees
    let flower-pollen count flowers with [ pollen2 >= 6 ]
    if ticks mod 51 = 50 [
; command occurs every 300 ticks (5 minutes)
        if flower-pollen >= bee_arrival_rate [
            hatch 1 [ set pollenV pollenV - pollenV
                    set pollenF pollenF - pollenF
                    ifelse random-float 1.01 < 1
                        [ set size 2 ]
                        [ set size 5 ]
            ]
        ]
    ]
end

to leave-bees
    let flower-pollen count flowers with [ pollen2 >= 6 ]
    if ticks mod 101 = 100 [
        if flower-pollen < 25 [ die ]
    ]
end

to bees-leave
    if energy > 10 or pollenV + pollenF > 45 [ die ]
end

to grow-veggies
    ask veggies [
        if pollen1 = tomato_growth and fruits < 7
        [ set pollen1 pollen1 - tomato_growth
          hatch-tomatoes 1 [
            rt random-float 360 fd 1
            set shape "circle"
            set color red
            set size 0.7
          ]
          set fruits fruits + 1
        ]
    ]
end

to create-pollen
    if ticks mod 901 = 900 [
        set pollen2 pollen2 + 1
    ]
end

```

```
]
end

to display-labels
  ask turtles [ set label "" ]
  if show-pollen? [
    ask flowers [ set label round pollen2 ]
    ask veggies [ set label round pollen1 ]
    ask bees [ set label round pollenV ]
  ]
end
```

APPENDIX B
OUTREACH ACTIVITY SLIDES

Simulating Bee-Plant Interactions





Morgan Beetler

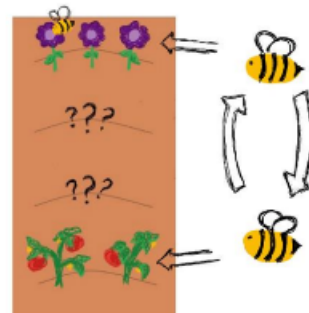


How does what you plant affect pollinators?



Background Information

-  Insects improve crop yields by providing pollination services
-  Amount of pollination services depends on the abundance of insect populations in the area
-  Local insect populations are affected by the management of agricultural landscapes and the natural resources that are available
-  We will explore the impact of incorporating flowers in vegetable gardens on bee activity and vegetable yields



Pollinator-Resource Model

Model Assumptions

- 1) Bees pollinate tomato plants
- 2) Bees pollinate flowers
- 3) Bees move freely between tomato plants and flowers
- 4) Tomato plants produce tomatoes due to pollination
- 5) Land can only accommodate so many tomatoes

Introduction: Understanding the Netlogo Simulation



Part I: Changing Ratio of Tomato Plants to Flowers

Develop a hypothesis: How do you think the ratio of tomato plants to flowers will affect the tomato yield? What makes you think that?

Instructions

- 1) Set the "percentage_tomato_plants" ratio bar to 50% tomato plants
- 2) Run the simulation, recording any observations.
- 3) Plot a graph showing the % Tomato Plants and the tomatoes the simulation yields for all ratios you recorded
- 4) Change the "percentage_tomato_plants" and repeat the process above

Discussion Questions

- What do you notice about the relationship between the % Tomato Plants and the tomatoes the simulation yields?
- Did your observations support your hypothesis? Why or why not?
- What do you notice about the graph of "Plant Yields"?
 - How would you describe the tomato growth? (Linear, quadratic, exponential, logarithmic, etc.)
 - What does the graph tell you about the relationship between Time and the # of Tomatoes?
- What do you notice about the bee activity in your garden?
- What conclusions can you draw from your findings?
- What questions arise from your findings?

Part II: Changing the Number of Bees

Develop a hypothesis: How do you think the number of bees will affect the tomato yield? What makes you think that?

Instructions

- 1) Set "initial-number-bees" to 1 bee
- 2) Run the simulation, recording any observations
- 3) Plot a graph showing the initial number of bees and the tomatoes the simulation yields for all simulations run
- 4) Change the "initial-number-bees" and repeat the process above



Part III: Finding Ideal Garden Conditions

We have separately explored the impact of the ratio of tomato plants to flowers planted and the initial number of bees on tomato yields.

How would these two variables work together to produce the best yield of tomatoes?

Instructions

- 1) Develop a plan for how you will investigate with your group. How do our findings from Part I and Part II influence where you will begin?
- 2) Create a hypothesis. What conditions will create the best crop yield?
- 3) Carry out your investigation.
- 4) What are your findings? What observations from the simulations support your claims?

Do you think you could achieve your highest tomato yield in an actual garden? Why?



Discussion Questions

- What do you notice about the relationship between the initial number of bees and the tomatoes the simulation yields?
- Did your observations support your hypothesis? Why or why not?
- What do you notice about the graph of "Plant Yields"?
 - How would you describe the tomato growth? (linear, quadratic, exponential, logarithmic, etc.)
 - What does the graph tell you about the relationship between Time and the # of Tomatoes?
- What do you notice about the bee activity in your garden?
- What conclusions can you draw from your findings?
- What questions arise from your findings?

Discussion Questions

- What is the relationship between initial number of bees, ratio of tomato plants to flowers planted, and tomato yields?
- What is the relationship between initial number of bees, ratio of tomato plants to flowers planted, and bee activity?
- What conclusions can you draw from your findings?
- What questions arise from your findings?

What other variables may influence tomato yields?

In our simulations, we explored tomato yields in relationship to bee population sizes and ratio of types of plants in our garden.

- What other variables could influence the pollination of tomato plants?



APPENDIX C
OUTREACH ACTIVITY STUDENT HANDOUT

Simulating Bee-Plant Interactions (Instructor's sheet)

Launch

Insects improve plant yields by providing pollination services, but how can we use this to our benefit? In this activity, we are going to explore the effects of plant varieties on pollinators and their benefits to yields.

You are tasked with planting a garden that produces the best tomato yields. There are many variables that you can change in your garden. These include:

- Ratio of tomato plants to flowers planted
- Initial number of bees
- Rate at which bees forage
- Units of pollen it takes to grow a tomato
- Rate at which bees enter your garden

By manipulating these variables, you can see their impact on your garden's tomato yields.

When determining a variable's impact on tomato yields, we will need to use skills we have learned in class. We will use data to plot, classify (linear, quadratic, exponential, etc.), and interpret graphs to draw conclusions. You will then use your conclusions to make decisions when designing your garden for the best tomato yields. You may choose to use additional math skills to help make informed decisions regarding your garden design. Whatever you use, make sure you find justification in your simulations.

Let's first explore two variables (one at a time) together, then you and your group will consider all five variables.

Activity Synthesis

Now that we have explored the bee-plant interactions in our gardens, how does what we plant affect pollinators?

Follow-up questions:

- What from your investigation makes you say that?
- Did everyone in your group come to the same conclusion? Why or why not?
- Does your conclusion match your hypothesis from the beginning of the activity? Why did it change?

All of you have a "highest" tomato yield from your investigation. On the handout I asked:

"How realistic is it that you would achieve that tomato yield in an actual garden? Why?"

What do we think? Can we achieve these tomato yields in our own gardens at home?

What are some take-aways from this experiment that we can apply when growing a garden?

Simulating Bee-Plant Interactions

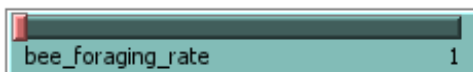
Activity Instructions

To plant your garden, you will use a Netlogo simulation. Use the steps below to access the simulation.

- 1) Download the file *_Bee_Simulation*
- 2) Go to <https://ccl.northwestern.edu/netlogo/> and click on *Go to Netlogo Web*
- 3) In the upper right corner of the screen by *Upload a Model* select *Choose File*
- 4) Select the file named *_Bee_Simulation*

PART I: Changing Ratio of Tomato Plants to Flowers

- 1) Here we are only interested in changing the ratio of tomato plants to flowers planted in our garden. Make sure all other variables are set accordingly:



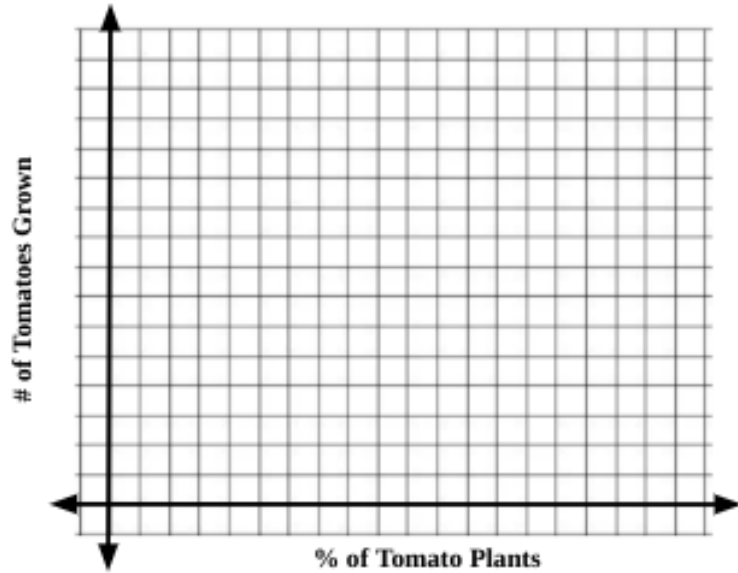
- 2) Develop a hypothesis: How do you think the ratio of tomato plants to flowers will affect the tomato yield? What makes you think that?
- 3) To start, set the “percentage_tomato_plants” as shown below:



- 4) Run the simulation and record observations:
 - a. What do you notice about the “Plant Yields” graph?
 - b. How would you describe tomato growth? (linear, quadratic, exponential, logarithmic, etc.)
 - c. What does the graph tell you about the relationship between time and the # of tomatoes?
 - d. What do you notice about the bee activity in your garden?

5) Record tomato yields in the table below, then plot the data on the graph provided.

% Tomato Plants	# of Tomatoes Grown



6) Repeat steps 4 & 5, only changing the “percentage_tomato_plants” to values of your choosing.

After you have collected your data, consider the following questions:

- ✓ What do you notice about the relationship between the % Tomato Plants and the tomatoes the simulation yields from your table and graph in step 5?
- ✓ Did your observations support your hypothesis? Why or why not?
- ✓ What conclusions can you draw from your findings?
- ✓ What questions arise from your findings?

PART II: Changing the Initial Number of Bees

1) Here we are only interested in changing the initial number of bees in our garden. Make sure all other variables are set accordingly:

percentage_tomato_plants 50 % tomato plants

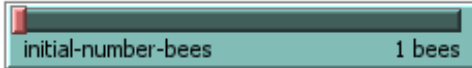
tomato_growth 4 units of pollen

bee_foraging_rate 1

bee_arrival_rate 15 or more plants with units of pollen

2) Develop a hypothesis: How do you think the initial number of bees will affect the tomato yield? What makes you think that?

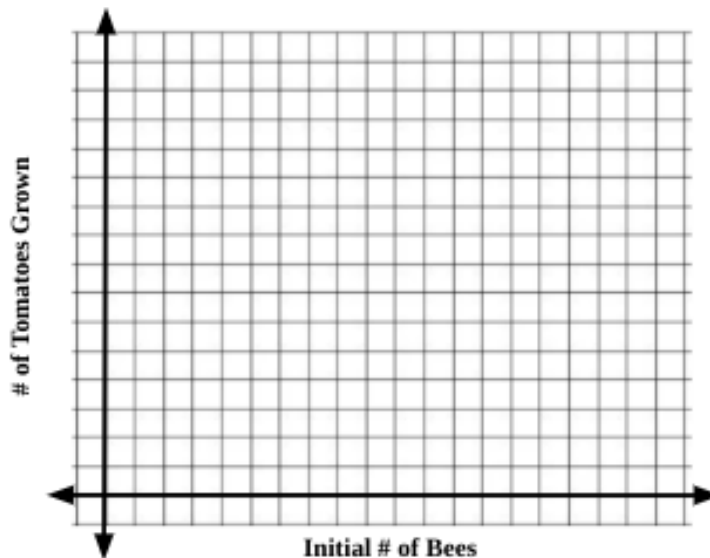
3) To start, set the “initial-number-bees” as shown below:



- 4) Run the simulation and record observations:
- What do you notice about the “Plant Yields” graph?
 - How would you describe tomato growth? (linear, quadratic, exponential, logarithmic, etc.)
 - What does the graph tell you about the relationship between time and the # of tomatoes?
 - What do you notice about the bee activity in your garden?

5) Record tomato yields in the table below, then plot the data on the graph provided.

Initial # of Bees	# of Tomatoes Grown



6) Repeat steps 4 & 5 only changing the “initial-number-bees” to values of your choosing.

After you have collected your data, consider the following questions:

- ✓ What do you notice about the relationship between the initial number of bees and the tomatoes the simulation yields from your table and graph in step 5?
- ✓ Did your observations support your hypothesis? Why or why not?
- ✓ What conclusions can you draw from your findings?
- ✓ What questions arise from your findings?

PART III: Finding Ideal Garden Conditions

So far, we have separately explored the impact of the ratio of tomato plants to flowers planted and the initial number of bees on tomato yields.

How would these two variables work together to produce the best tomato yields? What if we manipulated all five variables (Ratio of tomato plants to flowers planted, Initial number of bees, Rate at which bees forage, Units of pollen it takes to grow a tomato, and Rate at which bees enter your garden)?

Instructions

- 1) Develop a plan for how you will investigate with your group. How do our findings from Part I and Part II of the activity influence where you will begin?
- 2) Create a hypothesis. What conditions do you think will create the best tomato yield?
- 3) Carry out your investigation.
- 4) What are your findings? What observations from the simulations support your claims?

Highest Tomato Yield Achieved _____

Variable Settings to Achieve Tomato Yield:

percentage_tomato_plants	
initial-number-bees	
tomato_growth	
bee_foraging_rate	
bee_arrival_rate	

Briefly summarize the effects of each variable on tomato yields with justification that you found in your investigation. (About 1-2 sentences for each variable)

How realistic is it that you would achieve your tomato yield in an actual garden? Why or why not?

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BIOGRAPHICAL STATEMENT

Morgan N. Beetler received her B.S. degree in Mathematics from Iowa State University, Ames, Iowa, in 2020. Prior to pursuing her graduate degree at Texas Tech University, Morgan taught mathematics at Boone High School, Boone, Iowa from 2020-2022. Upon completion of her M.S. degree from Texas Tech University, she will continue her career in education and teach mathematics at Frenship High School, Wolfforth, Texas.