A New Block Preconditioner for Implicit Runge-Kutta Methods for Parabolic PDE
Md Masud Rana\textsuperscript{1}, Victoria E. Howle\textsuperscript{2}, Katharine Long\textsuperscript{3}, Ashley Meek\textsuperscript{4}, and William Milestone\textsuperscript{5}

\textbf{Abstract:} Explicit time integrators for parabolic PDE are subject to a restrictive time-step limit, so A-stable integrators are essential. It is well known that although there are no A-stable explicit linear multistep methods and implicit multistep methods cannot be A-stable beyond order two, there exist A-stable and L-stable implicit Runge-Kutta (IRK) methods at all orders. IRK methods offer an appealing combination of stability and high order; however, these methods are not widely used for PDE because they lead to large, strongly coupled linear systems. An \( s \)-stage IRK system has \( s \)-times as many degrees of freedom as the systems resulting from backward Euler or implicit trapezoidal rule discretization applied to the same equation set. In this talk, I will introduce a new block preconditioner for IRK methods, based on a block LDU factorization with algebraic multigrid subsolves for scalability. I will demonstrate the effectiveness of this preconditioner on the heat equation as a simple test problem, and compare in condition number and eigenvalue distribution, and in numerical experiments with other preconditioners currently in the literature. Experiments are run with IRK stages upto \( s = 7 \), and it is found that the new preconditioner outperforms the others, with the improvement becoming more pronounced as spatial discretization is refined and as temporal order is increased.

\textsuperscript{1} Texas Tech University (md-masud.rana@ttu.edu)
\textsuperscript{2} Texas Tech University (victoria.howle@ttu.edu)
\textsuperscript{3} Texas Tech University (katharine.long@ttu.edu)
\textsuperscript{4} Adams State University (ameek@adams.edu)
\textsuperscript{5} Texas Tech University (william.milestone@ttu.edu)