Outline

1. For $g:[a,b]\to\mathbb{C}$ define bounded variation, total variation $n(g)$

   Prop. $g:[a,b]\to\mathbb{C}$ piece-wise smooth $\Rightarrow g$ bounded variation & $n(g)=\int_a^b |g'(t)|\,dt$

2. For $f:[a,b]\to\mathbb{C}$, $g:[a,b]\to\mathbb{C}$ bounded variation define Riemann-Stieltjes integral

   Prop. $f:[a,b]\to\mathbb{C}$ continuous, $g:[a,b]\to\mathbb{C}$ bounded variation $\int_a^b f\,dg=\int_a^b f(t)dg(t)$

   exists

3. R-S Integrals linear & additive

   Prop. $f:[a,b]\to\mathbb{C}$ continuous, $g:[a,b]\to\mathbb{C}$ piece-wise smooth $\int_a^b f\,dg=\int_a^b f(t)\,g'(t)$

4. Define rectifiable path

   For $g:[a,b]\to\mathbb{C}$ bounded variation, $\{g\}\subset E$, $f:E\to\mathbb{C}$ continuous define line integral

   $$\int_g f=\int_a^b f\circ g\,dg=\int_a^b f\circ g(t)\,dg(t)=\int_g f(z)dz$$

   $$\left(=\int_a^b f(g(t))\,g'(d)dt \text{ for } g \text{ piece-wise smooth}\right)$$

5. Parameter invariance of line integral

   Define a curve as equivalance class of paths

6. Properties of line integrals

7. Prop. (Fund. Thm. of Calc for Line Integrals) G region, $g:[a,b]\to G$ rectifiable, $a=l(a)$, $b=g(b)$, $f:[a,b]\to\mathbb{C}$ continuous, f has primitive F on G

$$\Rightarrow \int_{g} f = F(b) - F(a)$$