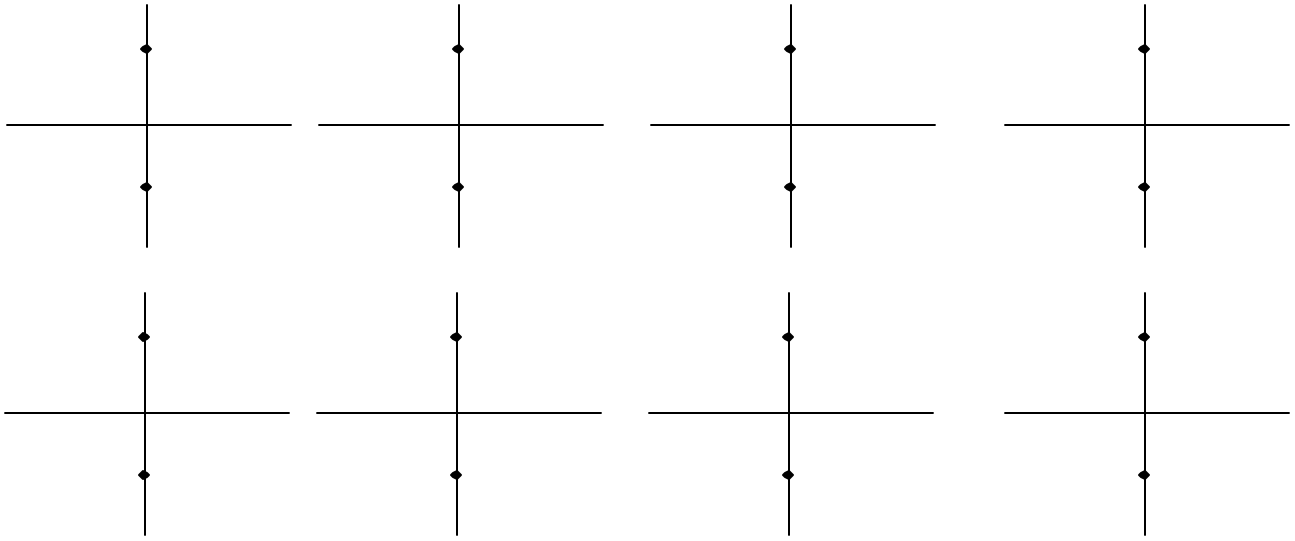
Math 4356 Exam III November 21, 2003

Answer the problems on separate paper. You do not need to rewrite the problem statements on your answer sheets. Do your own work. Show all relevant steps which lead to your solutions. Retain this question sheet for your records.

1. (10 pts) Find the value of $\int_{\Gamma} \frac{dz}{z^2 - 4z}$ where Γ is the circle $|z| = 2$ traversed once in the positive direction.

2. (10 pts) Find the value of $\int_{\Gamma} (z^2 - 4z)\,dz$ where $\Gamma = g_1 + g_2$ and g_1 is the line segment from 1 to $1+i$ and g_2 is the line segment from $1+i$ to i.

3. (8 pts) Consider the following eight contours $\Gamma_1, \Gamma_2, \cdots, \Gamma_8$ each of which lies in the doubly punctured plane $\mathbb{C} \setminus \{i, -i\}$. Identify which of the eight contours are homotopic to each other in $\mathbb{C} \setminus \{i, -i\}$.



4. (16 pts) Consider the following domains each of which is either the Right Half-Plane (RHP) with a subset of it omitted or the Right Half-Plane (RHP) with an additional set augmented to it. Determine which of these domains are simply connected. (Here D(z,r) denotes the closed disk centered at z of radius r.)

a. RHP \ (0,1] b. RHP \ [1,4) c. RHP \ [1,2]

d. RHP ∩ ann(1,1,2) e. RHP ∩ ann(1,2,3) f. RHP \ D(0,1)

g. RHP \ D(1,1) h. RHP \ D(2,1)

5. (6 pts) Explain why $e^{\frac{1-z^2}{1+z^2}}$ has an anti-derivative on the Right Half-Plane.

6. (20 pts) Find the value of the following integrals, where Γ is the circle $|z| = 2$ traversed once in the positive direction.

a. $\int_\Gamma \frac{ze^z}{3z-2}dz$ b. $\int_\Gamma \frac{2z+1}{(z+1)^2(z-3)}dz$

7. (8 pts) State the convergence properties of the Taylor series for:

a. $\frac{1}{1+z^2}$ at $z = 2$ b. $e^{-\sin z}$ at $z = 2$

8. (8 pts) Find the circle of convergence for the power series $\sum_{n=0}^{\infty} \frac{(z-i)^n}{2^n}$

9. (12 pts) For $f(z) = \sum_{n=0}^{\infty} (n+1)^2 z^n$ find

a. $f^{(4)}(0)$ b. $\int_\Gamma \frac{e^z f(z)}{z^2}dz$ where Γ is the circle $|z| = 2$ traversed once in the positive direction.

10. (12 pts) Find the Laurent series for $\frac{1}{z-z^2}$ on the annulus:

a. $0 < |z| < 1$ b. $0 < |z - 1| < 1$