MATH 3360-001 Exam III 15 November 2010

Make-UP

Section I. Answer the problems in this section on separate paper. You do not need to rewrite the problem statements on your answer sheets. Do your own work. Show all relevant steps which lead to your solutions.

1. Definitions

2. (15 pts) Prove the following proposition: Let H be a normal subgroup of G. If G is an abelian group, then G/H is an abelian group.

3. (10 pts) Give an example of each of the following (different from the examples given on page 207 in Papantonopoulou in Figure 1), if such an example exists – otherwise state that no such example exists:

a. A finite commutative ring with unity which is not an integral domain.
b. A commutative ring with no-zero divisors which is not an integral domain.
c. An infinite field.
d. An finite field which is not an integral domain.
e. A infinite integral domain which is not a field.

4. (15 pts) Let $S=\{\begin{bmatrix} a & 0 \\ b & -a \end{bmatrix}: a,b\in \mathbb{Z}\}$. Determine (provide appropriate relevant steps) whether S is a subring of $M(2,\mathbb{Z})$.

5. (15 pts) Determine (provide appropriate relevant steps) whether the map $\phi:\mathbb{Z}\times\mathbb{Z}\to\mathbb{Z}$ given by $\phi((a,b))=2a+3b$ is group homomorphism. If ϕ is a group homomorphism, then find Kern ϕ.

Name: ________________________ Score: ________________

Section II. Answer each of the following questions on this answer sheet. You do not need to provide rationales with your answers for the problems in this section. Staple this answer sheet to the front of your other pages.

6. (8 pts) Identify (Y/N) whether the following rings are integral domains:

a. ________ $\mathbb{Z}(\sqrt{7}) = \{a + b\sqrt{7} : a, b \in \mathbb{Z}\}$, usual real number addition and multiplication

b. ________ $\mathbb{Q}[i] = \{a + bi : a, b \in \mathbb{Q}, i^2 = -1\}$, usual complex number addition and multiplication

c. ________ $S = \{\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} : a, b \in \mathbb{R}\}$, usual $M(2, \mathbb{R})$ matrix addition and multiplication

d. ________ $\mathbb{Z}_p \times \mathbb{Z}_q$, where p, q are primes with $p \neq q$, usual direct product addition and multiplication

7. (8 pts) Identify (Y/N) whether the following rings are fields:

a. ________ $\mathbb{Q}(\sqrt{7}) = \{a + b\sqrt{7} : a, b \in \mathbb{Q}\}$, usual real number addition and multiplication

b. ________ $\mathbb{Z}[i] = \{a + bi : a, b \in \mathbb{Z}, i^2 = -1\}$, usual complex number addition and multiplication

c. ________ $S = \{\begin{bmatrix} a & 0 \\ b & a \end{bmatrix} : a \in \mathbb{Z}\}$, usual $M(2, \mathbb{Z})$ matrix addition and multiplication

d. ________ $\mathbb{Z}_2[i] = \{a + bi : a, b \in \mathbb{Z}_2, i^2 = -1\}$, usual $\mathbb{Z}_2$ addition and multiplication for sums and products of elements of $\mathbb{Z}_2$.

8. (10 pts) Identify (Y/N) whether the following quotient groups are abelian.
(10 pts) Identify (Y/N) whether the following quotient groups are cyclic.

a. Abelian ________ Cyclic _______ $\mathbb{Z}_{12} / < 8 >$

b. Abelian ________ Cyclic _______ $Sym(\square) / < \rho_0 >$, where $Sym(\square)$ is the symmetric group on the rectangle and ρ_0 is the identify map.

c. Abelian ________ Cyclic _______ S_5 / A_5

d. Abelian ________ Cyclic _______ $\mathbb{Z}_2 \times \mathbb{Z}_6 / < (1,4) >$

e. Abelian ________ Cyclic _______ $\mathbb{Z}_4 \times \mathbb{Z}_6 / < (2,4) >$