MATH 3360-001 | Exam I Make-Up | 29 September 2010

Answer the problems on separate paper. You do not need to rewrite the problem statements on your answer sheets. Do your own work. Show all relevant steps which lead to your solutions. For no question is True/Yes or False/No by itself sufficient as the answer. Retain this question sheet for your records.

1. Definitions

2. (12 pts) Consider the Eucidean plane $\mathbb{R}^2$. Determine whether the relation ~ defined on $\mathbb{R}^2$ by $(x_1, y_1) \sim (x_2, y_2)$ means $x_1 - x_2 = y_1 - y_2$ is an equivalence relation.

3. (12 pts) Use the Euclidean algorithm to find the greatest common divisor of 484 and 284.

4. (12 pts) Calculate the given expression and express the result in rectangular form:

a. $\frac{7-3i}{3-4i}$ b. $(-1+i)^{19}$

5. (12 pts) Calculate the following matrix computations

a. $\begin{bmatrix} 6 & 12 \\ 7 & 10 \end{bmatrix}\begin{bmatrix} 13 & 4 \\ 7 & 8 \end{bmatrix}$ in $M(2, \mathbb{Z}_{16})$ b. $\begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix}^7$ in $M(2, \mathbb{Z}_8)$

6. (24 pts) Consider the following sets G with associative operations *. In each case, determine whether (G, *) is a group. If (G,*) is not a group, state why it is not.

a. (8 pts) $G = \{\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} : a \in \mathbb{Q}\}$, * is non-standard matrix multiplication defined by $\begin{bmatrix} 1 & \alpha \\ 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & \beta \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & \alpha\beta \\ 0 & 1 \end{bmatrix}$ for $\begin{bmatrix} 1 & \alpha \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & \beta \\ 0 & 1 \end{bmatrix} \in G$.

b. (8 pts) $G = \{2^k : k \in \mathbb{Z}\}$, i.e., G is the set of all powers (positive, zero, negative) of 2, * is multiplication of real numbers

c. (8 pts) $G = \mathbb{R}^2$, * is non-standard multiplication defined by $(x_1, y_1) * (x_2, y_2) = (x_1 x_2, y_1 y_2)$ for $(x_1, y_1), (x_2, y_2) \in \mathbb{R}^2$.

7. (12 pts) Complete the following table so that $G = \{a, b, c, d\}$ with * is a commutative group:

*	a	b	c	d
a				
b				d
c				
d			b	

8. (12 pts) Determine whether the following sets *H* are subgroups of the given groups *G* with operations *

a. (6 pts) $H = \{a + b\sqrt{2} : a, b \in \mathbb{Q}\}^*$, i.e., *H* is the set of real numbers of the form $a + b\sqrt{2}$ where *a* and *b* are rational numbers not both zero; $G = \mathbb{R}^*$, i.e., *G* is the set on non-zero real numbers, * is multiplication of real numbers.

b. (6 pts) $H = \{\begin{bmatrix} a & b \\ c & d \end{bmatrix} : a, b, c, d \in \mathbb{Z}, a - b + c - d = 0\}$; $G = M(2, \mathbb{Z})$; * is matrix addition.