MATH 3360-001 Exam I June 13, 2007

Makeup

Answer the problems on separate paper. You do not need to rewrite the problem statements on your answer sheets. Do your own work. Show all relevant steps which lead to your solutions. For no question is True/Yes or False/No by itself sufficient as the answer. Retain this question sheet for your records.

1. Not repeatable

2. (12 pts) Consider the following sets *S* and equations, each of which defines a rule * for *S*. In each case, determine whether * defines an operation on *S*. If * does not define an operation on *S*, state why not.

1. $S = \left\{ \begin{bmatrix} a & b & 0 \\ c & d & e \\ 0 & f & g \end{bmatrix} \middle| a,b,c,d,e,f,g \in \mathbb{Z} \right\}$, * is matrix multiplication

2. *S* = set of positive odd integers, * is defined by the equation $m*n = \dfrac{m+n+2}{2}$

3. (12 pts) Consider the following equation, which defines an operation * on $\mathbb{Z}$:
$m*n = (m+n)mn$

a. Determine if * is associative. If not, state why not.
b. Determine if * has an identity. If not, state why not.
c. Determine if * is commutative. If not, state why not.

4. (12 pts) Complete the following Cayley table so that *G* = {*a*, *b*, *c*, *d*,*e*} with * is a commutative group:

*	a	b	c	d	e
a		c			
b				b	
c	e				
d					
e			b		

5. (18 pts) Consider the following sets G with rule *. In each case, determine whether (G, *) is a group. If (G,*) is not a group, state why it is not.

a. $G = \{2^n \mid n = 0, 1, 2, 3, 4, \cdots\}$, i.e., G is the set of non-negative powers of 2, * is multiplication of integers

b. $G = M(2,\mathbb{Z}) \setminus \{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\}$, i.e., G is the set of 2x2 matrices with integer entries minus the "zero matrix", * is multiplication of matrices

c. $G = M(\mathbb{Z})$, i.e., G is the set of mappings from $\mathbb{Z}$ to $\mathbb{Z}$, * is addition of functions

6. (12 pts) Write each of the following as a single cycle or as a product of disjoint cycles:

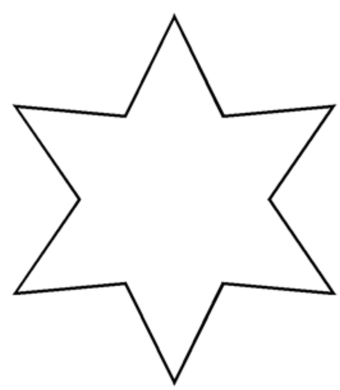
a. $\begin{pmatrix} 123456 \\ 364125 \end{pmatrix}$

b. (1 3 2 6) (3 5 1) (4 1 7 2)

c. $(2\ 6\ 3\ 1)^{-1}$ (1 3 5 2) (5 2 3)

7. (12 pts) It is known that $\mathbb{R}^2$ with component-wise addition (i.e., $(a,b)+(c,d)=(a+c,b+d)$) forms a group. Show that the subset $H = \{(x,y) \mid y = 2x,\ x, y \in \mathbb{R}\}$ is a subgroup of $\mathbb{R}^2$.

8. (12 pts) Identify the symmetry group of the figure to the right, i.e., construct a list (or a table) which identifies each of the elements which belong to the symmetry group of the figure. The figure is a regular 6-pointed sherrif's star.



Also, identify which elements in the symmetry group of the figure are self-inversive, i.e., which elements in the symmetry group of the figure are their own inverses.