MATH 1352-009 Exam I September 27, 2005

Answer the problems on separate paper. You do not need to rewrite the problem statements on your answer sheets. Work carefully. Do your own work. **Show all relevant supporting steps!**

1. (12 pts) Sketch the region bounded between the curves $y = x^3 + x^2 - 4x + 3$ and $y = 2x + 3$. Find the area of the bounded region.

2. Let $R$ be the region bounded between the curves $y = 2x$ and $y = x^2$. Set up, but do **not** evaluate, an integral to compute the volume of the solid of revolution generated by revolving the region $R$ about the indicated axis:

   a. (8 pts) the $y$-axis

   b. (8 pts) the line $y = -2$

3. (12 pts) Find all of the points of intersection between the polar curves $r = 3\cos\theta$ and $r = 1 + \cos\theta$.

4. (16 pts) The polar curves $r = 2 - 2\cos\theta$ and $r = 2$ intersect at the symmetric points $(2, \pi/2)$ and $(2, -\pi/2)$. Find the area of the region that is inside $r = 2$ and outside $r = 2 - 2\cos\theta$.

5. (12 pts) Setup, but do **not** evaluate, an integral to find the length of the curve $x^2 = 4y^4$ from the point (0,0) to (8,2).

6. (16 pts) Seawater in the Giant Ocean tank at the New England Aquarium is continuously pumped so that the seawater in the entire tank is filtered and recycled every 4 hours. Normally, the seawater level in the tank is kept at a constant total depth of 12 meters. A spiral ramp goes from ground level to the top of the tank with observation windows along the way so that viewers can see different "sea layers". The observation windows are all the same dimension: 2 meter high by 3 meters wide. How much more force is exerted by the seawater in the tank on a observation window near the bottom of the tank as compared to a window near the top of tank? Specifically, the observation windows near the bottom of the tank are placed so that the top of the windows are 9 meters below the surface of the seawater and the observation windows near the top of the tank are placed so that the top of the windows are 1 meter below the surface of the seawater. Recall that the density constant for seawater is 62.0.

7. (16 pts) Find the $\bar{x}$ coordinate of the centroid of the region in the first quadrant bounded by the curves $y = \sqrt[3]{x}$ and $y = x^3$.

   Bonus (4 pts) Find the $\bar{y}$ coordinate of the centroid of the region in the first quadrant bounded by the curves given in Problem 7.