

Answer the problems on separate paper. You do not need to rewrite the problem statements on your answer sheets. Do your own work. Show all relevant steps which lead to your solutions. Retain this question sheet for your records.

Part I. Do each problem (Each problem is worth 7 points)

1. Find $\frac{dy}{dx}$ given that $x^3 - 2xy^2 = x - 1$.
2. Find $\frac{dy}{dx}$ given that $y = x^2 \sin^{-1}(1 - x)$.
3. An electric winch is sitting on a dock (at the end) and is reeling in a cable at the rate of 18 ft/min. The dock is 12 feet above the water level. The end of the cable is attached to an eye bolt on the bow (front edge) of a row boat. The eye bolt is 2 feet above the water line of the row boat. How fast is the row boat moving towards the dock at the moment that the bow of the row boat is 24 feet from the end of the dock?
4. Use the linearization of $y = e^{-x}$ at $x = 0$ to approximate the value of $e^{-0.05}$.
5. Find the minimum and maximum values of the function $y = 2x^3 + 3x^2 - 12x + 4$ on the interval $[-1, 4]$.
6. My son needs to make a closed rectangular box (with a square base) in which to display an ant colony for his school science project. The materials to make the sides of the box and the bottom of the box will cost $\$0.04 / \text{in}^2$ while the clear plastic for the top of the box will cost $\$0.005 / \text{in}^2$. If the requirements of his science project are that the box should provide 600 cubic inches of space for the ant colony, will he be able to make the box for less than $\$15.00$? If so, what dimensions (to the nearest tenth of an inch) should he choose for the making the box to minimize the cost? What will that minimum cost be?
7. Find the limit, if it exists, $\lim_{x \rightarrow 0} \frac{1 - \cos^2 2x}{\sin x^2}$.
8. Find the limit, if it exists, $\lim_{x \rightarrow 0^+} \frac{1}{x} - \frac{1}{e^x - 1}$.
9. Find $\int 8(u + \frac{1}{u}) du$
10. Find $\int (3x - 5)^2 dx$
11. Find $\int \frac{t^2 + t - 1}{\sqrt{t}} dt$

Part II. Do one of the following problems (at 23 points).

1. For $y = f(x) = \frac{x^2 - 4x - 5}{x + 3} = x - 7 + \frac{16}{x + 3}$ identify:
- any intervals on which f is increasing
 - any intervals on which f is decreasing
 - any intervals on which f is concave up
 - any intervals on which f is concave down
 - any points on the graph of f which are relative maximums of f
 - any points on the graph of f which are relative minimums of f
 - any points on the graph of f which are inflection points of f
 - any points on the graph of f which are intercepts of the graph of f
 - any asymptotes to the graph of f

Incorporate all of the above into a sketch of the graph of f

2. For a certain function $y = f(x)$ it can be easily verified using Maple that

$f'(x) = -200 \frac{x^3 - 6x^2 - 8}{x^2(x^2 + 4)^2}$ and that the numerator $x^3 - 6x^2 - 8$ only has one root (at approximately at 6.2).

It can be easily verified using Maple that

$f''(x) = 200 \frac{3x^5 - 4x^3 - 24x^4 - 48x^2 - 64}{x^3(x^2 + 4)^3}$ and that the numerator $3x^5 - 4x^3 - 24x^4 - 48x^2 - 64$ only has one root (at approximately 8.4).

Furthermore, an analysis shows that the line $x = 0$ is a vertical asymptote to the graph of f and that the line $y = 0$ is a horizontal asymptote to the graph of f and that the graph of f has no other asymptotes.

Finally, it can be shown that the graph of f has an x-intercept at (4,0) and no other intercepts.

Identify:

- any intervals on which f is increasing
- any intervals on which f is decreasing
- any intervals on which f is concave up
- any intervals on which f is concave down
- determine whether the graph of f has any points which are relative maximums of f
- determine whether the graph of f has any points which are relative minimums of f
- determine whether the graph of f has any points which are inflection points of f

Using all of the above information generate a sketch for what the graph of f should look like.