Mathematics 2450, Calculus 3 with applications

Fall 2012, Version A

The use of calculator, formula sheet and/or any other electronic device is not allowed.

Multiple choice questions.

Follow the directions of the instructor.

1. Find the **parametric** equations for the line passing through the point $P = (1, 2, 3)$ and perpendicular to the plane $-2x - y + 2z = 1$.

   a) $t\mathbf{i} + 2t\mathbf{j} + 3t\mathbf{k}$ b) $< -2 + t, -1 + 2t, 2 + 3t >$

   c) $\dfrac{x+2}{1} = \dfrac{y+1}{2} = \dfrac{z-2}{3}$ d) $x + 2y + 3z = 0$

   e) $< 1 - 2t, 2 - t, 3 + 2t >$

2. Let the velocity vector be $\mathbf{v}(t) = e^t\mathbf{i} - \sin(2t)\mathbf{j} + t^2\mathbf{k}$, and the initial position vector be $\mathbf{r}(0) = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$. Compute the position vector $\mathbf{r}(t)$.

   a) $< e^t, -2\cos(2t), 2t >$ b) $< e^t + 1, \dfrac{1}{2}\cos(2t) + \dfrac{1}{2}, \dfrac{1}{3}t^3 - 1 >$

   c) $< e^t, \dfrac{1}{2}\cos(2t), \dfrac{1}{3}t^3 >$ d) $< e^t + 1, -2\cos(2t) + 3, 2t - 1 >$

   e) $< 2, 1, -1 >$

3. Given $F(x, y) = \cos(xy)$ where $x = u^2 + v^2$ and $y = u^2 - v^2$. Use the chain rule (do not substitute for x and y!) to find $\dfrac{\partial F}{\partial v}$. Express the result in terms of x, y, u, and v.

   a) $\dfrac{\partial F}{\partial v} = -\sin(xy)(y - x)2v$ b) $\dfrac{\partial F}{\partial v} = -\sin(xy)(y + x)2u$

   c) The function is not differentiable d) $\dfrac{\partial F}{\partial v} = -\sin(xy)(2yu - 2xv)$

   e) $\dfrac{\partial F}{\partial v} = -\sin(xy)(2yv + 2xu)$

4. Let $f(x, y) = \cos(x + 3y)$, $P = (\pi/2, \pi/3)$ and $\mathbf{v} = -3\mathbf{i} + 4\mathbf{j}$. Find the directional derivative of f at P in the direction of $\mathbf{v}$.

   a) 9 b) 0

   c) $\dfrac{9}{5}$ d) $\dfrac{1}{5} < -3, 4 >$

   e) $< 1, 3 >$

5. Find the equation of the plane tangent to the graph $z = 5 - x^2 - 3y^2$ at $(x, y) = (0, 1)$.

a) $x + y + z = 1$ b) $z = 2$
c) $2x^2 + 6y^2 - 6y + z - 2 = 0$ d) $6y + z = 8$
e) $x = 0$

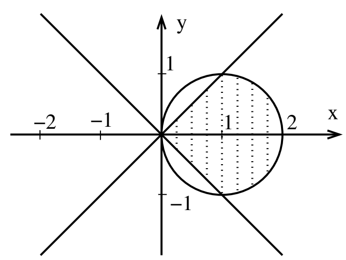
6. Reverse the order of integration for the following integral

$$I = \int_0^1 \int_y^1 f(x, y)\, dx dy.$$

a) $\int_y^1 \int_0^1 f(x, y)\, dy\, dx$ b) $\int_1^0 \int_1^y f(x, y)\, dx\, dy$
c) $\int_0^1 \int_0^x f(x, y)\, dy\, dx$ d) $\int_0^1 \int_x^1 f(x, y)\, dy\, dx$
a) $\int_y^1 \int_0^1 f(y, x)\, dy\, dx$

7. Set up the double integral in polar coordinates (do not compute it!) for finding the area of the region in the plane defined by

$$D : \begin{cases} -x \le y \le x \\ (x-1)^2 + y^2 \le 1 \end{cases}$$



a) $\int_{\theta=-\pi/4}^{\theta=\pi/4} \int_{r=0}^{r=1} dr\, d\theta$ b) $\int_{\theta=-\pi/2}^{\theta=\pi/2} \int_{r=0}^{r=1} r\, dr\, d\theta$
c) $\int_{\theta=-\pi/4}^{\theta=\pi/4} \int_{r=0}^{r=2\cos\theta} r\, dr\, d\theta$ d) $\int_{\theta=-\pi/4}^{\theta=\pi/4} \int_{r=0}^{r=2\cos\theta} dr\, d\theta$
e) $\int_{\theta=-\pi/2}^{\theta=\pi/2} \int_{r=0}^{r=2\sin\theta} r\, dr\, d\theta$

8. Find the surface area of the portion of the cone $z = \sqrt{x^2 + y^2}$ inside the cylinder $x^2 + y^2 = 4$.

a) $S = 4\sqrt{2}\,\pi$ b) $S = 0$
c) $S = 4\pi$ d) $S = \frac{\pi}{6}(17^{3/2} - 1)$
f) $S = \pi$

9. Find curl $\mathbf{F}$, where

$$\mathbf{F}(x, y, z) = \cos(x)\mathbf{i} + e^{y^2}\mathbf{j} + (z^3 + 2z)\mathbf{k}.$$

a) $\nabla \times \mathbf{F} = \sqrt{\sin^2(x) + \left(2e^{y^2}\right)^2 + (3z^2+2)^2}$ b) $\nabla \times \mathbf{F} = 0$
c) $\nabla \times \mathbf{F} = -\sin(x) + 2e^{y^2} + 3z^2 + 2$ d) $\nabla \times \mathbf{F} =< -\sin(x), 2e^{y^2}, 3z^2 + 2 >$
e) $\nabla \times \mathbf{F} =< 0, 0, 0 >$

10. Verify if the vector field $\mathbf{F} =< x\cos(2y), -x^2\sin(2y) >$ is conservative and evaluate the line integral

$$I = \int_C \mathbf{F} \cdot d\mathbf{R},$$

where C is the curve parametrized by $\mathbf{R} =< t, \pi t^2 >$, for $0 \le t \le 1$.

a) $I = \frac{1}{4}$ b) $I = \frac{1}{2}$
c) $I = 0$ d) $I = 1$
e) $I = 2$

11. Use Green's theorem to evaluate

$$I = \oint_C \left(-y + y^2\right) dx + (x + 2xy)\, dy,$$

where C is the rectangle with vertices in $(0,0)$, $(2,0)$, $(2,1)$ and $(0,1)$, traversed counterclockwise.

a) $I = 4$ b) $I = 0$
c) $I = 1$ d) $I = 2$
e) $I = 8$

12. Use the divergence theorem to evaluate

$$I = \iint_S \mathbf{F} \cdot \mathbf{N}\, dS,$$

where $\mathbf{F} =< z\sin(y), e^x + y, z + \cos(xy) >$, and $\mathbf{N}$ is the unit outward normal to the surface S defined implicitly by

$$x^2 + y^2 + z^2 = 1.$$

a) $I = 0$ b) $I = \pi$
c) $I = 1$ d) $I = \frac{8}{3}\pi$
e) $I = 4\pi$

Essay questions.

Show all your work. A correct answer with no work counts as 0.

13. Let the position vector be $\mathbf{R}(t) = 8t\,\mathbf{i} + 3\sin(2t)\,\mathbf{j} - 3\cos(2t)\mathbf{k}$. Find the unit tangent vector $\mathbf{T}(t)$ and the principal unit normal vector $\mathbf{N}(t)$.

14. Find the absolute extrema for the function $z = f(x, y) = x\,y$, in the closed disk $x^2 + y^2 \leq 1$.

15. Use either cylindrical or spherical coordinates to evaluate the triple integral

$$I = \iiint_{\mathbf{D}} z\,dV,$$

where $\mathbf{D}$ is the portion of the ball, $x^2 + y^2 + z^2 \leq 4$, in the first octant, $x \geq 0,\ y \geq 0$ and $z \geq 0$.

16. Use Stokes' theorem to evaluate the line integral $\oint_C \mathbf{F} \cdot d\mathbf{R}$, where

$$\mathbf{F} = (xe^x - y^2)\mathbf{i} + (\cos(y^2) - xy)\mathbf{j} + z^2\mathbf{k},$$

and C is the closed curve given by the line segments connecting the points $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$ traversed in the given order.