1. Find the parametric equations for the line passing through the point \( P = (1, 2, 3) \) and perpendicular to the plane \(-2x - y + 2z = 1\).

a) \( t\mathbf{i} + 2t\mathbf{j} + 3t\mathbf{k} \)  
   b) \( < -2 + t, -1 + 2t, 2 + 3t > \)
   c) \( \frac{x + 2}{1} = \frac{y + 1}{2} = \frac{z - 2}{3} \)  
   d) \( x + 2y + 3z = 0 \)
   e) \( < 1 - 2t, -1 + 2t, 3 + 2t > \)

2. Let the velocity vector be \( \mathbf{v}(t) = e^t\mathbf{i} - \sin(2t)\mathbf{j} + t^2\mathbf{k} \), and the initial position vector be \( \mathbf{r}(0) = 2\mathbf{i} + \mathbf{j} - \mathbf{k} \). Compute the position vector \( \mathbf{r}(t) \).

a) \( < e^t, -2\cos(2t), 2t > \)  
   b) \( < e^t + 1, \frac{1}{2}\cos(2t) + \frac{1}{2}, \frac{1}{3}t^3 - 1 > \)
   c) \( < e^t, \frac{1}{2}\cos(2t), \frac{1}{3}t^3 > \)  
   d) \( < e^t + 1, -2\cos(2t) + 3, 2t - 1 > \)
   e) \( < 2, 1, -1 > \)

3. Given \( F(x, y) = \cos(xy) \) where \( x = u^2 + v^2 \) and \( y = u^2 - v^2 \). Use the chain rule (do not substitute for \( x \) and \( y \)!) to find \( \frac{\partial F}{\partial v} \). Express the result in terms of \( x, y, u, \) and \( v \).

a) \( \frac{\partial F}{\partial v} = -\sin(xy)(y - x)2v \)  
   b) \( \frac{\partial F}{\partial v} = -\sin(xy)(y + x)2u \)
   c) The function is not differentiable  
   d) \( \frac{\partial F}{\partial v} = -\sin(xy)(2yu - 2xv) \)
   e) \( \frac{\partial F}{\partial v} = -\sin(xy)(2yu + 2xv) \)

4. Let \( f(x, y) = \cos(x + 3y) \), \( P = (\pi/2, \pi/3) \) and \( \mathbf{v} = -3\mathbf{i} + 4\mathbf{j} \). Find the directional derivative of \( f \) at \( P \) in the direction of \( \mathbf{v} \).

a) \( 9 \)  
   b) \( 0 \)
   c) \( \frac{9}{5} \)  
   d) \( \frac{1}{5} < -3, 4 > \)
   e) \( < 1, 3 > \)
5. Find the equation of the plane tangent to the graph \( z = 5 - x^2 - 3y^2 \) at \((x, y) = (0, 1)\).

   a) \( x + y + z = 1 \)  
   b) \( z = 2 \)  
   c) \( 2x^2 + 6y^2 - 6y + z - 2 = 0 \)  
   d) \( 6y + z = 8 \)  
   e) \( x = 0 \)

6. Reverse the order of integration for the following integral

   \[ I = \int_0^1 \int_y^1 f(x, y) \, dx \, dy. \]

   a) \( \int_y^1 \int_0^1 f(x, y) \, dy \, dx \)  
   b) \( \int_0^y \int_0^1 f(x, y) \, dx \, dy \)  
   c) \( \int_0^1 \int_0^x f(x, y) \, dy \, dx \)  
   d) \( \int_0^1 \int_0^y f(x, y) \, dy \, dx \)  
   e) \( \int_y^1 \int_0^1 f(y, x) \, dy \, dx \)

7. Set up the double integral in polar coordinates (do not compute it!) for finding the area of the region in the plane defined by

   \[ D: \begin{cases} 
   -x \leq y \leq x \\ 
   (x - 1)^2 + y^2 \leq 1
   \end{cases} \]

   a) \( \int_{\theta=-\pi/4}^{\theta=\pi/4} \int_{r=0}^{r=1} dr \, d\theta \)  
   b) \( \int_{\theta=-\pi/2}^{\theta=\pi/2} \int_{r=0}^{r=1} r \, dr \, d\theta \)  
   c) \( \int_{\theta=-\pi/4}^{\theta=\pi/4} \int_{r=0}^{r=2 \cos \theta} r \, dr \, d\theta \)  
   d) \( \int_{\theta=-\pi/4}^{\theta=\pi/4} \int_{r=0}^{r=2 \cos \theta} dr \, d\theta \)  
   e) \( \int_{\theta=-\pi/2}^{\theta=\pi/2} \int_{r=0}^{r=2 \sin \theta} r \, dr \, d\theta \)  

8. Find the surface area of the portion of the cone \( z = \sqrt{x^2 + y^2} \) inside the cylinder \( x^2 + y^2 = 4 \).

   a) \( S = 4\sqrt{2} \pi \)  
   b) \( S = 0 \)  
   c) \( S = 4\pi \)  
   d) \( S = \frac{\pi}{6} (17^{3/2} - 1) \)  
   f) \( S = \pi \)
9. Find curl $F$, where

$$F(x, y, z) = \cos(x)i + e^{y^2}j + (z^3 + 2z)k.$$ 

a) $\nabla \times F = \sqrt{\sin^2(x) + (2e^{y^2})^2 + (3z^2 + 2)^2}$

b) $\nabla \times F = 0$

c) $\nabla \times F = -\sin(x) + 2e^{y^2} + 3z^2 + 2$

d) $\nabla \times F = \langle -\sin(x), 2e^{y^2}, 3z^2 + 2 \rangle$

e) $\nabla \times F = \langle 0, 0, 0 \rangle$

10. Verify if the vector field $F = \langle x \cos(2y), -x^2 \sin(2y) \rangle$ is conservative and evaluate the line integral

$$I = \int_C F \cdot d\mathbf{R},$$

where $C$ is the curve parametrized by $\mathbf{R} = \langle t, \pi t^2 \rangle$, for $0 \leq t \leq 1$.

a) $I = \frac{1}{4}$

b) $I = \frac{1}{2}$

c) $I = 0$

d) $I = 1$

e) $I = 2$

11. Use Green’s theorem to evaluate

$$I = \oint_C (-y + y^2) \, dx + (x + 2xy) \, dy,$$

where $C$ is the rectangle with vertices in $(0, 0), (2, 0), (2, 1)$ and $(0, 1)$, traversed counterclockwise.

a) $I = 4$

b) $I = 0$

c) $I = 1$

d) $I = 2$

e) $I = 8$

12. Use the divergence theorem to evaluate

$$I = \iiint_S F \cdot \mathbf{N} \, dS,$$

where $F = \langle z \sin(y), e^x + y, z + \cos(xy) \rangle$, and $\mathbf{N}$ is the unit outward normal to the surface $S$ defined implicitly by

$$x^2 + y^2 + z^2 = 1.$$ 

a) $I = 0$

b) $I = \pi$

c) $I = 1$

d) $I = \frac{8}{3}\pi$

e) $I = 4\pi$
Essay questions.

Show all your work. A correct answer with no work counts as 0.

13. Let the position vector be \( \mathbf{R}(t) = 8t \mathbf{i} + 3\sin(2t) \mathbf{j} - 3\cos(2t) \mathbf{k} \). Find the unit tangent vector \( \mathbf{T}(t) \) and the principal unit normal vector \( \mathbf{N}(t) \).

14. Find the absolute extrema for the function \( z = f(x, y) = xy \), in the closed disk \( x^2 + y^2 \leq 1 \).

15. Use either cylindrical or spherical coordinates to evaluate the triple integral

\[
I = \iiint_D z \, dV,
\]

where \( D \) is the portion of the ball, \( x^2 + y^2 + z^2 \leq 4 \), in the first octant, \( x \geq 0, \ y \geq 0 \) and \( z \geq 0 \).

16. Use Stokes’ theorem to evaluate the line integral \( \oint_C \mathbf{F} \cdot d\mathbf{R} \), where

\[
\mathbf{F} = (xe^x - y^2) \mathbf{i} + (\cos(y^2) - xy) \mathbf{j} + z^2 \mathbf{k},
\]

and \( C \) is the closed curve given by the line segments connecting the points \( (1, 0, 0) \), \( (0, 1, 0) \), \( (0, 0, 1) \) traversed in the given order.