Show all work and clearly indicate final answers in your blue books.

1. Solve exactly (no calculator approximations): \( x^2 + 4x + 1 = 0. \)

2. Find the center and radius for the circle given by \( 2x^2 + 4x + 2y^2 - 8y + 8 = 0. \)

3. Solve the inequality: \( \frac{x + 1}{2x - 1} < 0. \)

4. Let \( f(x) = \frac{2}{x + 1}. \) Find \( f^{-1}(x) \) and verify your answer by calculating \( f(f^{-1}(x)). \)

5. The price of a new machine is $31,500. After 15 years, the machine has a salvage value of $1,500. Assuming linear depreciation, find a formula for the value of the machine after \( t \) years, \( 0 \leq t \leq 15. \)

6. Determine the following for the function \( f(x) = \frac{x - 1}{x^2 - 2x}. \)
   
   a) Domain of \( f \)
   
   b) Range of \( f \)
   
   c) \( x \)-intercepts (if any)
   
   d) \( y \)-intercepts (if any)
   
   e) vertical asymptotes
   
   f) Sketch the graph of \( y = f(x). \)

7. Solve for \( x: \) \( \log_2(x) - \log_2(x - 2) = 1. \)

8. The amount of the radioactive element sodium-24 in a sample after \( t \) hours is given by \( N = N_0e^{-0.047t}, \) where \( N_0 \) is the current amount. If there are currently 2 g of sodium-24 in the sample, how long before there is only 1 g?

9. Give exact values for the following expressions (not calculator approximations). If the answer is in an angle, use radians.
   
   a) \( \sin\left(\frac{5\pi}{3}\right) \)
   
   b) \( \cos(-120^\circ) \)
   
   c) \( \tan\left(\frac{5\pi}{6}\right) \)
   
   d) \( \sin^{-1}\left(\frac{1}{2}\right) \)
   
   e) \( \sin\left[\cos^{-1}\left(\frac{5}{13}\right)\right] \).
10. Given that \( \tan \theta = \frac{2}{3} \) and \( \theta \) is in the third quadrant, find exact values for the other 5 trig functions of \( \theta \).

11. Graph at least one period of \( y = 2 \sin(3x - \pi) \). State clearly the amplitude, period, phase shift, and intercepts.

12. Find all solutions on the interval \( [0, 2\pi) \) of \( 2 \sin^2 x - \sin x = 0 \).

13. To an observer standing 500 feet from the base of a radio tower, the angle of elevation of the top of the tower is 36°. How high is the tower to the nearest foot.

14. A triangle has sides of \( a = 5 \), \( b = 4 \), and \( c = 6 \). (All measurements are given in feet.) Determine angle \( A \) of the triangle in degrees to one decimal place.

15. Solve the system

\[
\begin{align*}
2x - y &= -5 \\
x + 2y &= 5.
\end{align*}
\]

16. The polar coordinates of a point are \( (2, \frac{5\pi}{6}) \). Find the rectangular coordinates for the point.

17. Let \( \mathbf{a} = \langle 1, 1 \rangle \) and \( \mathbf{b} = \langle 3, 4 \rangle \). Find

a) \( \mathbf{a} + \mathbf{b} \)

b) \( |\mathbf{b}| \).

18. Find the partial fraction decomposition:

\[
\frac{x}{(x + 2)(x - 3)}.
\]