Math 1550 Final Exam – Fall 2011

Show all of your work in your blue book, or you will lose credit. Work the problems in order, and use one page per problem. Be neat, and use proper notation. Write out the formulas you use.

1. Sketch the graph of \( f(x) = \frac{6-x}{3-x} \). Also, state the following clearly:
   a) Domain of \( f \)
   b) Range of \( f \)
   c) Intercept(s) with x-axis
   d) Intercept(s) with y-axis
   e) Vertical asymptote(s)
   f) Horizontal asymptote(s)

2. Find the inverse function of \( f(x) = \frac{x}{2} - 4 \). Check your answer by calculating \( f(f^{-1}(x)) \). Then graph the function and the inverse on the same set of axes. Label clearly the graphs and their intercepts with the axes.

3. A sunflower is 15 inches tall after 7 days, and 36 inches tall after 14 days.
   a) Assume that the growth is linear, and find an equation that relates the height in inches \( y \) to the age in days \( x \).
   b) Use the equation to predict the height after 18 days.

4. Find the center and radius for the circle given by \( 4x^2 + 16x + 4y^2 - 32y - 64 = 0 \).

5. Solve for \( x \): \( \frac{x-5}{x-8} \geq 0 \)

6. A ball is thrown straight up, and its height as a function of time is given by \( h = -16t^2 + 96t \), where \( h \) is in feet and \( t \) is in seconds. Find the maximum height the ball reaches, and how long it takes to hit the ground.

7. Solve for \( x \): \( \log_4(x) + \log_4(x-6) = 2 \)

8. Recall the continuous compound interest formula \( A = Pe^{rt} \). A $5,000 deposit is made at 6% interest, compounded continuously. How long will it be until there is $8,000 in the account?

9. Graph at least one period of \( y = 3\sin(x - \pi / 2) \).
   State clearly the period, intercepts, phase shift, and amplitude.

(Over)
10. Give the *exact* values for the following expressions (*not* calculator approximations):

a) \( \sin (240^\circ) \)

b) \( \cos (- \pi/4) \)

c) \( \sin^2(15^\circ) + \cos^2(15^\circ) \)

d) \( \cos^{-1}(\sqrt{3}/2) \)

e) \( \tan[ \cos^{-1}(5/13) ] \)

11. An angle \( \theta \) is in the third quadrant and its tangent is \( (3/4) \). Find the *exact* values of the other 5 trig functions (*not* calculator approximations).

12. Find all solutions on the interval \([0, 2\pi)\):

\[
\sin^2 x + 3 \sin x + 2 = 0
\]

13. Prove the identity:

\[
\frac{\sin \theta + \cos \theta}{\sec \theta + \csc \theta} = \sin \theta \cos \theta
\]

14. From the top of a lighthouse, the angle of depression of a ship at sea is \( 5^\circ \). The ship is 1800 feet from the base of the lighthouse. How high is the lighthouse (to the nearest foot)?

15. Two of the angles of a triangle are \( 42^\circ \) and \( 68^\circ \). The side opposite the \( 42^\circ \) angle is 11 inches long. How long is the side opposite the \( 68^\circ \) angle?

16. Two of the sides of a triangle are 7 and 10 inches, and the included angle (the angle between them) is \( 55^\circ \). How long is the other side?

17. Solve the system:

\[
\begin{align*}
3x - 5y &= -33 \\
2x + 4y &= 66
\end{align*}
\]

18. Sketch the graph:

\[
\frac{(x - 5)^2}{25} + \frac{(y - 3)^2}{9} = 1
\]

State the name of the figure. Label clearly on the graph the center, the foci, and the intercepts with the x and y axes.

19. Find all of the roots (zeros):

\[
x^3 - 5x^2 - 2x + 24 = 0
\]

20. Find the partial fraction decomposition:

\[
\frac{2x + 15}{x(x^2 + 3)}
\]