Math 1452 Final Exam Spring 2013

1. Consider the region bounded by $y = x^2$, $x = 2$, and the x-axis. **Set up** (but do not solve) integrals to find

   (a) The volume of the solid generated by rotating this region about the y-axis using shells.

   (b) The volume of the solid generated by rotating this region about the horizontal line $y = -3$ using any method.

   (c) The moment about the x-axis of this region.

2. Graph the rose $r = 5\cos(2\theta)$ and find the area of one leaf.

3. Evaluate the following integrals.

   (a) $\displaystyle\int \sin^3(x)\, dx$

   (b) $\displaystyle\int x\,\frac{e^x + e^{-x}}{2}\, dx$

   (c) $\displaystyle\int \frac{\sin(3x)}{1 + \cos^2(3x)}\, dx$

   (d) $\displaystyle\int \frac{5x - 5}{(x-2)(x+3)}\, dx$

4. Evaluate $\displaystyle\int_3^\infty x e^{-x^2}\, dx$.

5. Indicate if the following series converge or diverge. You must identify all the tests you use and show all the work needed to apply them.

   (a) $\displaystyle\sum_{k=1}^\infty \frac{3 + \ln k}{k}$

   (b) $\displaystyle\sum_{k=1}^\infty \frac{3^k}{k\, 2^k}$

   (c) $\displaystyle\sum_{k=2}^\infty \frac{2k - 3}{k^3 + 2}$

   (d) $\displaystyle\sum_{k=0}^\infty \frac{3^k}{k!}$

6. Does the series $\displaystyle\sum_{k=1}^\infty \frac{(-2)^k}{3^k}$ converge? If it converges, find the sum. If not, explain why not.

7. Find the first 3 terms of the Taylor series for $f(x) = x \ln x$ about $x = 3$.

8. If $\mathbf{u} = <0, 2, -1>$ and $\mathbf{v} = <3, 1, 0>$, find

   (a) $\| \mathbf{u} - 2\mathbf{v} \|$

   (b) $\mathbf{u} \times \mathbf{v}$

   (c) The cosine of the angle between $\mathbf{u}$ and $\mathbf{v}$