Math 1452 Final Exam Spring 2013

Calculators are not allowed on this exam. Work all questions completely. Show all work as described in class.
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1. Consider the region bounded by \( y = x^2, \) \( x = 2, \) and the \( x \)-axis. Set up (but do not solve) integrals to find
   (a) The volume of the solid generated by rotating this region about the \( y \)-axis using shells.
   (b) The volume of the solid generated by rotating this region about the horizontal line \( y = -3 \) using any method.
   (c) The moment about the \( x \)-axis of this region.

2. Graph the rose \( r = 5 \cos(2\theta) \) and find the area of one leaf.

3. Evaluate the following integrals.
   (a) \( \int \sin^3(x) \, dx \)
   (b) \( \int x \frac{e^x + e^{-x}}{2} \, dx \)
   (c) \( \int \frac{\sin(3x)}{1 + \cos^2(3x)} \, dx \)
   (d) \( \int \frac{5x - 5}{(x - 2)(x + 3)} \, dx \)

4. Evaluate \( \int_{-\infty}^{\infty} x e^{-x^2} \, dx. \)

5. Indicate if the following series converge or diverge. You must identify all the tests you use and show all the work needed to apply them.
   (a) \( \sum_{k=1}^{\infty} \frac{3 + \ln k}{k} \)
   (b) \( \sum_{k=1}^{\infty} \frac{3^k}{k^{2k}} \)
   (c) \( \sum_{k=2}^{\infty} \frac{2k - 3}{k^3 + 2} \)
   (d) \( \sum_{k=0}^{\infty} \frac{3^k}{k!} \)

6. Does the series \( \sum_{k=1}^{\infty} \frac{(-2)^k}{3^k} \) converge? If it converges, find the sum. If not, explain why not.

7. Find the first 3 terms of the Taylor series for \( f(x) = x \ln x \) about \( x = 3. \)

8. If \( \mathbf{u} = \langle 0, 2, -1 \rangle \) and \( \mathbf{v} = \langle 3, 1, 0 \rangle, \) find
   (a) \( \| \mathbf{u} - 2\mathbf{v} \| \)
   (b) \( \mathbf{u} \times \mathbf{v} \)
   (c) The cosine of the angle between \( \mathbf{u} \) and \( \mathbf{v} \)