1. The GMAT scores (the test required to enter a graduate school business program) and the undergraduate GPA for 6 students are given below. Find the slope and intercept of the best fit line for the following data using a linear regression.

<table>
<thead>
<tr>
<th>GMAT</th>
<th>750</th>
<th>620</th>
<th>800</th>
<th>720</th>
<th>530</th>
<th>680</th>
</tr>
</thead>
<tbody>
<tr>
<td>GPA</td>
<td>3.9</td>
<td>3.1</td>
<td>3.95</td>
<td>3.2</td>
<td>2.1</td>
<td>3.5</td>
</tr>
</tbody>
</table>

2. The profit (in millions of dollars) associated with manufacturing \( x \) units of a certain commodity is given by

\[
P(x) = \frac{5x^2 - 3x + 200}{4x^2 + 50}
\]

What is the profit as number of units increases without bound? That is, evaluate the limit of the profit as \( x \) tends to infinity.

3. A company manufactures a series of 52-in flat screen televisions. The quantity \( x \) of these sets (in thousands) demanded each week is related to the wholesale unit price \( p \) by the following equation

\[
p = 700 - 20x
\]

The weekly total cost incurred by the company for producing \( x \) sets is represented by the following equation

\[
C(x) = 50x + 450
\]

a) Find the revenue function \( R(x) \).

b) Find the profit function \( P(x) \).

c) Find the marginal profit function.

4. Find the marginal revenue function for the given price function

\[
p(x) = \ln(3 - \sqrt{x})
\]

5. Find the marginal cost function for the given cost function

\[
C(x) = \frac{e^{3x^4}}{x^5 + 3x}
\]

6. The percentage of households using Netflix \( t \) years after 2000 is given by

\[
f(t) = 2t^3 - t + 5
\]

How fast was the percentage of Netflix users changing in 2003?

7. The monthly cost for renting \( x \) apartments is given by the function

\[
C(x) = x^3 - 51x^2 + 40000
\]

How many units should the complex rent to minimize their cost? What is the minimum cost the complex can realize?

8. The quantity demanded each month for Texas Tech sports apparel is related to the unit price given by

\[
p = 40 - 2x, \quad \text{where} \quad p \text{ is measured in dollars and} \ x \text{ is measured in thousands.} \]

To yield the maximum revenue, how many items must be sold? How much should they charge?

9. Katie wants to make gift boxes for her Thanksgiving guest. She decides to make her box out of 8 in by 8 in card stock by cutting equal squares out of the corners and folding up the remaining sides. What size does she need to cut the squares to make the volume a maximum? What are the dimensions of the box? What is the volume of the box?
10. After analyzing the data, Leonard determines that the total annual revenue $R$ brought in by his computer manufacturing company is related to the amount of money $x$ spent on new research and development by the function

$$R(x) = x^4 - 10x^3 + 36x^2 + 12x + 24$$

where both $R$ and $x$ are measured in thousands of dollars. Find the company's point of diminishing returns on its research spending (that is, find where the marginal revenue is zero; i.e., find the inflection point.).

11. Evaluate the following integrals
   a)
   $$\int \frac{(\ln x)^4}{x} \, dx$$
   b)
   $$\int_0^1 (2x + 3)\sqrt{x^2 + 3x + 4} \, dx$$
   c)
   $$\int 4e^x - \frac{4}{x^2} + 1 \, dx$$

12. Based on a preliminary report of a geological survey team, it is estimated that a newly discovered oil field can be expected to produce oil at a rate of

$$R'(x) = \frac{x^2 - 2}{x^3 - 6x + 8}$$

millions barrels/year, $x$ year after production begins. Find the amount of oil that the field can be expected to yield during the 5 year of production, assuming that the projection holds true.

13. Find the area of the region under the graph of the function $f(x) = \frac{2}{x} + x^2$ on the interval $[1,3]$.

14. Gus just bought a 5-year franchise license for a doughnut shop that he expects will generate income at a rate of

$$R(t) = \text{300,000}$$

dollars each year. If the prevailing interest rate over the 5 years is 8% per year compounded continuously, find the present value of the franchise.

15. The demand function, $p = -0.01x^2 - 0.04x + 7.6$, for a brand of certain printer ink replacement cartridges demanded per week, in thousands. Determine the consumers' surplus if the market price is set at $6/cartridge.

### Formulas for Math 1331

<table>
<thead>
<tr>
<th>Formula</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1}{b-a} \int_a^b f(x) , dx )</td>
<td>Average Value</td>
</tr>
<tr>
<td>( L = 2\int_0^x [x - f(x)] , dx )</td>
<td></td>
</tr>
<tr>
<td>( CS = \int_0^x D(x) , dx - \bar{p}\bar{x} )</td>
<td></td>
</tr>
<tr>
<td>( PS = \bar{p}\bar{x} - \int_0^x S(x) , dx )</td>
<td>Consumers' Surplus</td>
</tr>
<tr>
<td>( A = \frac{mP}{r} (e^{rt} - 1) )</td>
<td>Amount at time $t$</td>
</tr>
<tr>
<td>( PV = \frac{mP}{r} (1 - e^{-rt}) )</td>
<td>Present Value</td>
</tr>
<tr>
<td>( A = \int_0^t R(t) e^{-rt} , dt )</td>
<td></td>
</tr>
</tbody>
</table>
