Final Exam
MATH 1300
Spring 2013

Instructions: Solve 14 of the problems 1-16. If you solve more than 14, you must clearly mark which 14 you want to be graded. For full credit, you must show complete, correct, legible work. Read carefully before you start working. No books or notes are allowed. Calculators are allowed; phones and PDAs are not.

1.) Construct Pascal's triangle and use it to answer the following questions.

   (a) If a family has 6 children, in how many ways could there be 3 boys and 3 girls?
   (b) In how many ways can a person arrange 2 dogs and 5 cats in a straight line?

2.) Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and define $A = \{1, 2, 4, 5, 8\}$, $B = \{2, 3, 7, 8, 9\}$, and $C = \{6\}$. Find the following.

   (a) $\overline{B \cup C}$
   (b) $(A \cup B) \cap C$
   (c) $|A \cap B|$

3.) State whether the following arguments are valid or invalid.

   (a) If I win the race, I will get a gold medal.
       I will be happy if I get a gold medal.
       Therefore, if I win the race, I will be happy.

   (b) If Susan does not study, she will fail the exam.
       Susan did not fail the exam.
       Therefore, Susan studied.

   (c) If you win the lottery, then you will quit your job.
       You quit your job.
       Therefore, you won the lottery.

4.) Use Kruskal's Algorithm to find a minimum spanning tree for the graph below. If the given weights represent hundreds of dollars, what is the cost of the minimum spanning tree?

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A     6     B     3     C
5          2    7          12
D     4     E     8     F
9          5     3        4
G     7     H     5     I
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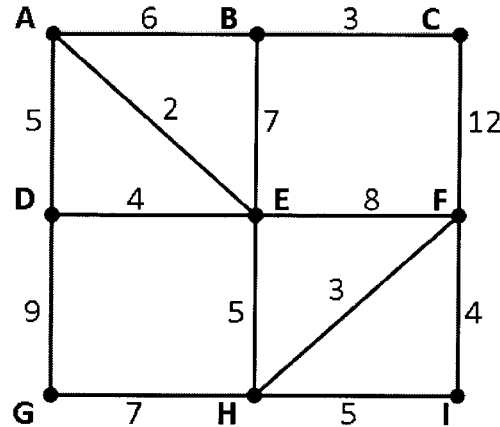
5.) Consider a triangle with the following measurements:
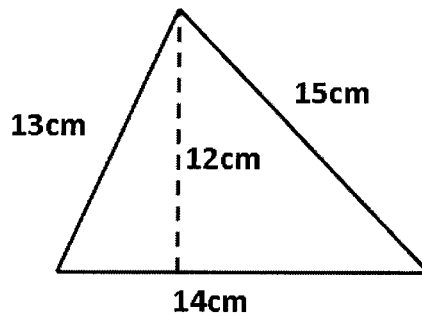
13cm   15cm
        12cm
        14cm

    (a) Find the perimeter.
    (b) Find the area.
    (c) Imagine that the figure is the base of a 3-dimensional object that comes straight out of the page, forming a triangular prism that stands 8 cm tall. Find the volume of this 3-D figure.

6.) Solve each equation for x.

    (a) $\ln(3x + 1) = \ln(16)$ (b) $\log(x) = 2$

7.) You invest $3,500 into an account paying 3% annual interest compounded monthly. If you make no other transactions, how much money will be in your account after 15 years?

8.) In a ranked voting between 3 candidates, the outcomes were:

(ABC)	(ACB)	(BAC)	(BCA)	(CAB)	(CBA)
4	2	0	10	6	3

Determine the winner, if any, using each of the following voting methods. Clearly label what work and answer goes with each method.

    (a) Majority Method
    (b) Plurality Method
    (c) Borda Count Method
    (d) Hare Method

9.) State whether the following argument is valid or invalid. Justify your answer using Euler circles.

    All college students drink coffee.
    Most college students are hard workers.
    Therefore, some people who work hard drink coffee.

10.) In a recent survey of 100 people, the following information was gathered:

    51 people own Item A
    57 people own Item B
    35 people own Item C
    16 people own Item A and Item B
    19 people own Item A and Item C
    18 people own Item B and Item C
    7 people own Item A, Item B, and Item C

Draw and fill in a Venn diagram representing this information, and use it to answer the following questions.

    (a) How many people own both Item A and Item B, but not Item C?
    (b) How many people own exactly one of these items?

11.) Is the following statement a tautology? Justify your answer by constructing a truth table.

$$\sim(p \wedge q) \to (\sim q \vee p)$$

12.) Use the following network to answer the questions below.

      (a) Does the network have an Euler circuit?

      (b) Does the network have a Hamiltonian cycle?
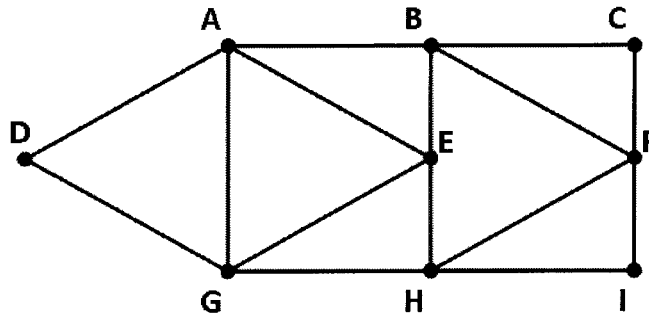
13.) Consider a circle with a radius that measures 3 inches.

      (a) Find the perimeter.

      (b) Find the area.

      (c) Imagine that the figure is the base of a 3-dimensional object that comes out of the page to a point, forming a circular cone that stands 5 inches tall. Find the volume of this 3-D figure.

14.) In 2010, an African safari wildlife park had a giraffe population of 40 giraffes. Assuming an annual growth rate of 1.3%, in what year should you expect the park to have 50 giraffes?

15.) The Smiths want to buy a $200,000 house and have $35,000 set aside to use as a down payment. If they finance the remaining balance with a 30-year loan that charges 6% interest compounded monthly, what will be the amount of their monthly payments?

16.) A town has the following district populations:

North: 7,600   South: 18,200   East: 2,400   West: 9,300

Suppose there are exactly 100 city council seats. Apportion the 100 seats out to each district according to Hamilton's Plan.

Future Value Formula (Simple Interest)

$$A = P(1 + rt)$$

Future Value Formula (Compound Interest)

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Future Value Formula (Continuous Compounding)

The future value, A, of an investment of P, *compounded continuously* at a rate of r for t years, is found by

$$A = Pe^{rt}$$

Amortization Formula

If the amount of the loan is known (P), and you wish to find the amount of the periodic payment (m), use the formula

$$m = \frac{P\left(\frac{r}{n}\right)}{1 - \left(1 + \frac{r}{n}\right)^{-nt}}$$

where r is the annual rate, t is the time (in years), and n is the number of payments per year.

Ordinary Annuity Formula

STOP This is one of the most useful formulas for you to use in your personal financial planning.

The future value, A, of an annuity is found with the formula

$$A = m\left[\frac{\left(1 + \frac{r}{n}\right)^{nt} - 1}{\frac{r}{n}}\right]$$

where r is the annual rate, m the periodic payment, t the time (in years), and n the number of payments per year.

Present Value of an Annuity

If the periodic payment is known (m) and you wish to find the present value of those periodic payments, use the *present value of an annuity* formula:

$$P = m\left[\frac{1 - \left(1 + \frac{r}{n}\right)^{-nt}}{\frac{r}{n}}\right]$$

where P is the present value of the annuity, r is the annual interest rate, and n is the number of payments per year.

Sinking Fund Formula

Spend a few minutes with this idea; in your own words, can you explain when you would use this formula?

If the future value (A) is known, and you wish to find the amount of the periodic payment (m), use the *sinking fund formula*

$$m = \frac{A\left(\frac{r}{n}\right)}{\left(1 + \frac{r}{n}\right)^{nt} - 1}$$

where r is the annual rate, t is the time (in years), and n is the number of times per year the payments are made.

Growth/Decay Formula

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This is one of the most useful formulas from mathematics.

> Exponential **growth** or **decay** can be described by the equation
>
> $$A = A_0 e^{rt}$$
>
> where r is the annual growth/decay rate, t is the time (in years), A_0 is the amount present initially (present value), and A is the target (future) value. If r is positive, this formula models growth, and if r is negative, the formula models decay.

TABLE 17.2 Summary of Voting Methods

Method	Description
Majority Method	Each voter votes for one candidate. If the number of voters is n and n is even, then the candidate with $\frac{n}{2} + 1$ or more votes wins. If the number n is odd, then the candidate with $\frac{n+1}{2}$ or more votes wins.
Plurality Method	Each voter votes for one candidate. The candidate receiving the most votes wins.
Borda Count Method	Each voter ranks the candidates. Each last-place candidate is given 1 point, each next-to-last candidate is given 2 points, and so on. The candidate with the highest number of points wins.
Hare Method	Each voter votes for one candidate. If a candidate receives a majority of the votes, that candidate is the winner. If no candidate receives a majority, eliminate the candidate with the fewest first-place votes and repeat the process until there is a majority candidate, who wins.
Pairwise Comparison Method	Each voter ranks the candidates. Each candidate is compared to each of the other candidates. If choice A is preferred to choice B, then A receives 1 point. If B is preferred to A, then B receives one point. If the candidates tie, then each receives $\frac{1}{2}$ point. The candidate with the most points wins.
Tournament Method	This method compares the entire slate of candidates two at a time, in a predetermined order. The first and second candidates are compared, the candidate with the fewer votes is eliminated, and the winner is then compared with the third candidate. These pairwise comparisons continue until the final pairing, which selects the winner.
Approval Method	Each voter casts one vote for all the candidates that meet with his or her approval. The candidate with the most votes is declared the winner.

TABLE 17.9 Summary of Apportionment Methods

Method	Divisor	Apportionment
Adams' Plan	Round **up; raise** the standard divisor to find the modified divisor.	Round the standard quotas up. Apportion to each group its modified upper quota. It favors the smaller states.
Jefferson's Plan	Round **down; lower** the standard divisor to find the modified divisor.	Round the standard quotas down. Apportion to each group its modified lower quota. It favors the larger states.
Hamilton's Plan	Use the standard divisor. Round **down.**	Round the standard quotas down. Distribute additional seats one at a time until all items are distributed.
Webster's Plan	Use modified divisors. May round up or down.	Round by comparing with the **arithmetic mean** of the upper and lower quotas.
HH's Plan	Use modified divisors. May round up or down.	Round by comparing with the **geometric mean** of the upper and lower quotas.