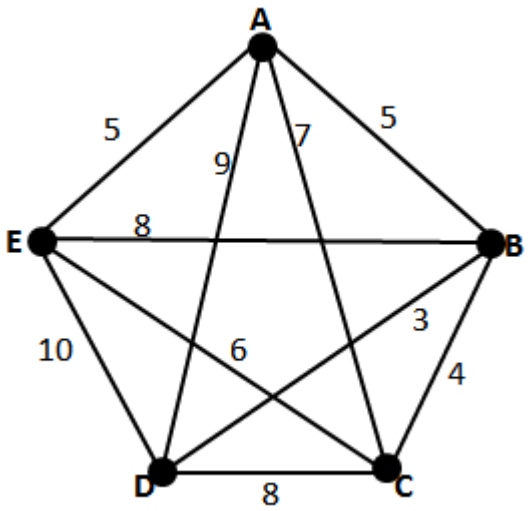
MATH 1300 ©
FINAL EXAM DEPARTMENT OF MATH AND STAT
FALL 2013 TEXAS TECH UNIVERSITY

Instructions: Solve 13 of the problems 1–15. If you solve more than 13 problems, you must clearly mark which 13 you want to have graded. For full credit, you must show complete, correct, legible work. Read carefully before you start working. No books or notes are allowed. Calculators are allowed, phones and PDAs are not.

1. Use the best edge algorithm to find a Hamilton circuit of minimal weight in the graph



2. The students in Hutchinson Middle School are preparing a charity event. A 13-person steering committee oversees the planning; it has representatives of 5^{th}, 6^{th}, 7^{th}, and 8^{th} graders in proportion to the number of students in each grade. Given that there are 123 5^{th}-graders, 105 6^{th}-graders, 95 7^{th}-graders, and 106 8^{th}-graders, apportion the 13 seats on the committee using Hamilton's method.

3. Use Euler diagrams to decide if each of the following syllogisms is valid.

	Some children love cookies		All men are animals
	<u>Dani does not love cookies</u>		<u>All men are rational</u>
$\therefore$	Dani is not a child	$\therefore$	Some animals are rational

4. If a single card is drawn from a standard 52-card deck, what is the probability that it is either a five or a red card?

5. The city of Portland, Oregon has a mean of 155 rainy days per year. Assuming a normal distribution and a standard deviation of 12 days per year, what percent of years would we expect for Portland to have:

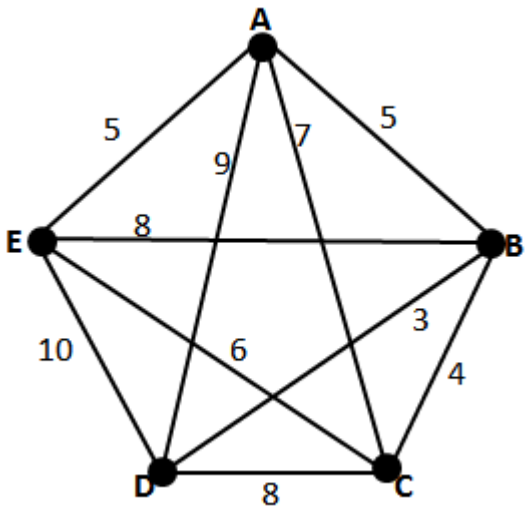
   (a) At least 164 rainy days?

   (b) At most 182 rainy days?

MATH 1300 ©
FINAL EXAM DEPARTMENT OF MATH AND STAT
FALL 2013 TEXAS TECH UNIVERSITY

Instructions: Solve 13 of the problems 1–15. If you solve more than 13 problems, you must clearly mark which 13 you want to have graded. For full credit, you must show complete, correct, legible work. Read carefully before you start working. No books or notes are allowed. Calculators are allowed, phones and PDAs are not.

1. Use the best edge algorithm to find a Hamilton circuit of minimal weight in the graph



2. The students in Hutchinson Middle School are preparing a charity event. A 13-person steering committee oversees the planning; it has representatives of 5^{th}, 6^{th}, 7^{th}, and 8^{th} graders in proportion to the number of students in each grade. Given that there are 123 5^{th}-graders, 105 6^{th}-graders, 95 7^{th}-graders, and 106 8^{th}-graders, apportion the 13 seats on the committee using Hamilton's method.

3. Use Euler diagrams to decide if each of the following syllogisms is valid.

	Some children love cookies		All men are animals
	<u>Dani does not love cookies</u>		<u>All men are rational</u>
$\therefore$	Dani is not a child	$\therefore$	Some animals are rational

4. If a single card is drawn from a standard 52-card deck, what is the probability that it is either a five or a red card?

5. The city of Portland, Oregon has a mean of 155 rainy days per year. Assuming a normal distribution and a standard deviation of 12 days per year, what percent of years would we expect for Portland to have:

   (a) At least 164 rainy days?

   (b) At most 182 rainy days?

6. Increasing the down payment on a mortgage reduces both the size of the monthly payments and the total interest paid. The Alaya family purchases a house that costs $\$155,000$. They finance it with a 20-year mortgage with an annual interest rate of 8%. To answer the following questions, you can use the table below.

   (a) Find the monthly payment assuming that the Alaya's make a down payment of $\$39,000$.

   (b) Find the monthly payment assuming that the Alaya's make a down payment of $\$49,000$.

   (c) How much less will the Alaya's pay in total if they make the bigger down payment?

Monthly payments on a $1,000 loan.

Annual Interest Rate	Number of Years for the Loan				
	3	4	10	20	30
4%	$29.53	$22.58	$10.12	$6.06	$4.77
5%	29.97	23.03	10.61	6.60	5.37
6%	30.42	23.49	11.10	7.16	6.00
8%	31.34	24.41	12.13	8.36	7.34
10%	32.27	25.36	13.22	9.65	8.78
12%	33.21	26.33	14.35	11.01	10.29

7. State A has a population of 1.7 million people and 14 representatives, state B has a population of 3.9 million people and 31 representatives, and state C has a population of 2.3 million people and 20 representatives.

   (a) Calculate the Huntington–Hill number for each state.

   (b) According to the Huntington–Hill apportionment method, which of the three states is the most deserving of an additional representative?

8. In an election between three candidates, A, B, and C, the voters have the following preferences:

First choice	A	A	B	B	C
Second choice	B	C	C	A	A
Third choice	C	B	A	C	B
Number of voters	6	21	18	23	12

   (a) Is any candidate preferred by the majority of the voters?

   (b) Which candidate wins if the election is decided by Borda count?

   (c) Which candidate wins if the election is decided by pairwise comparison?.

9. Construct a truth table for the statement

$$(\sim p \wedge q) \rightarrow \sim q\,.$$

10. A simple game involves a player rolling two dice and observing the result. If the sum is less than 4, then the player wins \$8; if the sum is greater than 9, then the player wins \$4; otherwise, the player loses \$2. What is the expected value of this game?

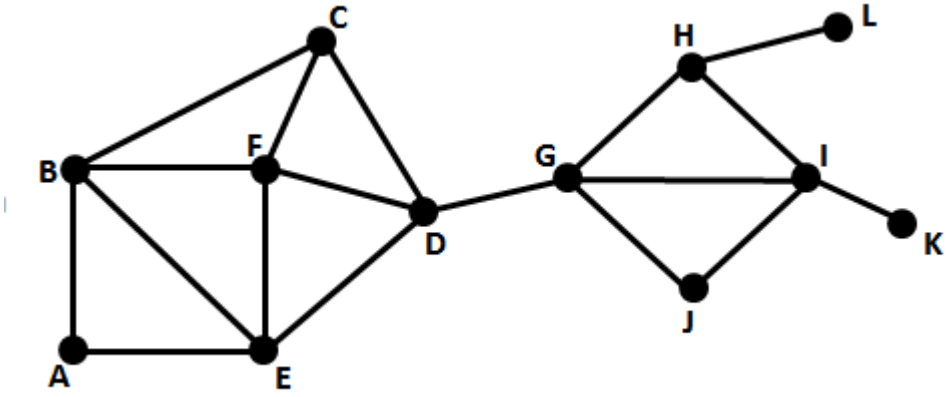
11. Decide if the two described operations give the same result for all numbers x and y.

    (a) Add x squared and y squared
    vs.
    Add x and y, and then square.

    (b) Add x squared and y squared, and then add 2 times x times y
    vs.
    Add x and y and 0, and then square.

12. Alexandra bought a new car for \$22,500. She made a \$3,000 down payment and financed the remaining amount by securing a 5 year loan with an annual interest rate of 7.2% compounded monthly. Find the amount of Alexandra's monthly payment.

13. Consider the graph



    (a) How many edges does the graph have?

    (b) Which vertices are odd and which vertices are even?

    (c) Is the graph connected?

    (d) Does the graph have any bridges? If so, identify one.

    (e) Can the graph be traced? If not, explain what part of Euler's theorem fails.

14. Consider the weighted voting system

$$[61 : 50, 10, 1, 60] .$$

    (a) Find all the winning coalitions and determine the critical voters in each winning coalition; you can call the voters A, B, C, and D.

    (b) Is it fair to say that the voter with 50 votes is more powerful than the one with 1 vote?

15. The annual salaries of the TTU football coaches are

1,850,000
425,000
240,000
235,000
215,000
210,000
210,000
210,000
190,000
175,000
90,000

(a) Find the mean salary.

(b) Find the median salary.

(c) Make a box plot to illustrate the salary distribution.

Method	How the Winning Candidate Is Determined
Plurality	The candidate receiving the most votes wins.
Borda count	Voters rank all candidates by assigning a set number of points to first choice, second choice, third choice, and so on; the candidate with the most points wins.
Plurality-with-elimination	Successive rounds of elections are held, with the candidate receiving the fewest votes being dropped from the ballot each time, until one candidate receives a majority of votes.
Pairwise comparison	Candidates are compared in pairs, with a point being assigned the voters' preference in each pair. (In the case of a tie, each candidate gets a half point.) After all pairs of candidates have been compared, the candidate receiving the most points wins.

TABLE 11.8 Summary of voting methods.

HAMILTON'S APPORTIONMENT METHOD

a) Find the standard divisor for the apportionment (total population/total number of representatives).

b) Find the standard quota (state's population/standard divisor) for each state and round it down to its lower quota. Assign that number of representatives to each state.

c) If there are any representatives left over, assign them to states in order according to the size of the fractional parts of the states' standard quotas.

ADAMS'S APPORTIONMENT METHOD

a) Use trial and error to find a modified divisor that is larger than the standard divisor for the apportionment.

b) Calculate the modified quota (state's population/modified divisor) for each state and round it up. Assign that number of representatives to each state. (Keep varying the modified divisor until the sum of these assignments is equal to the total number being apportioned.)

THE HUNTINGTON–HILL APPORTIONMENT PRINCIPLE

If states X and Y have already been allotted x and y representatives, respectively, then state X should be given an additional representative in preference to state Y provided that

$$\frac{(\text{population of Y})^2}{y \cdot (y+1)} < \frac{(\text{population of X})^2}{x \cdot (x+1)}$$

Otherwise, state Y should be given the additional representative. We will often refer to a number of the form $\frac{(\text{population of X})^2}{x \cdot (x+1)}$ as a **Huntington–Hill number.**

RULE FOR COMPUTING THE PROBABILITY OF A UNION OF TWO EVENTS If E and F are events, then

$$P(E \cup F) = P(E) + P(F) - P(E \cap F).$$

If E and F have no outcomes in common, they are called *mutually exclusive events.* In this case, because $E \cap F = \varnothing$, the preceding formula simplifies to

$$P(E \cup F) = P(E) + P(F).$$

GENERAL RULE FOR COMPUTING $P(F|E)$ If E and F are events in a sample space, then $P(F|E) = \dfrac{P(E \cap F)}{P(E)}$.

FORMULA FOR FINDING PAYMENTS ON AN AMORTIZED LOAN Assume that you borrow an amount P, which you will repay by taking out an amortized loan. You will make m periodic payments per year for n total payments and the annual interest rate is r. Then, you can find your payment by solving for R in the equation

$$P\left(1 + \frac{r}{m}\right)^n = R\left(\frac{\left(1 + \frac{r}{m}\right)^n - 1}{\frac{r}{m}}\right).^*$$

FORMULA FOR FINDING THE FUTURE VALUE OF AN ORDINARY ANNUITY Assume that we are making n regular payments, R, into an ordinary annuity. The interest is being compounded m times a year and deposits are made at the end of each compounding period. The future value (or amount), A, of this annuity at the end of the n periods is given by the equation

$$A = R\frac{\left(1 + \frac{r}{m}\right)^n - 1}{\frac{r}{m}}.$$

FORMULA FOR CONVERTING RAW SCORES TO z-SCORES Assume a normal distribution has a mean of μ and a standard deviation of σ. We use the equation

$$z = \frac{x - \mu}{\sigma}$$

to convert a value x in the nonstandard distribution to a z-score.

z	A	z	A	z	A	z	A	z	A	z	A
.00	.000	.56	.212	1.12	.369	1.68	.454	2.24	.488	2.80	.497
.01	.004	.57	.216	1.13	.371	1.69	.455	2.25	.488	2.81	.498
.02	.008	.58	.219	1.14	.373	1.70	.455	2.26	.488	2.82	.498
.03	.012	.59	.222	1.15	.375	1.71	.456	2.27	.488	2.83	.498
.04	.016	.60	.226	1.16	.377	1.72	.457	2.28	.489	2.84	.498
.05	.020	.61	.229	1.17	.379	1.73	.458	2.29	.489	2.85	.498
.06	.024	.62	.232	1.18	.381	1.74	.459	2.30	.489	2.86	.498
.07	.028	.63	.236	1.19	.383	1.75	.460	2.31	.490	2.87	.498
.08	.032	.64	.239	1.20	.385	1.76	.461	2.32	.490	2.88	.498
.09	.036	.65	.242	1.21	.387	1.77	.462	2.33	.490	2.89	.498
.10	.040	.66	.245	1.22	.389	1.78	.463	2.34	.490	2.90	.498
.11	.044	.67	.249	1.23	.391	1.79	.463	2.35	.491	2.91	.498
.12	.048	.68	.252	1.24	.393	1.80	.464	2.36	.491	2.92	.498
.13	.052	.69	.255	1.25	.394	1.81	.465	2.37	.491	2.93	.498
.14	.056	.70	.258	1.26	.396	1.82	.466	2.38	.491	2.94	.498
.15	.060	.71	.261	1.27	.398	1.83	.466	2.39	.492	2.95	.498
.16	.064	.72	.264	1.28	.400	1.84	.467	2.40	.492	2.96	.499
.17	.068	.73	.267	1.29	.402	1.85	.468	2.41	.492	2.97	.499
.18	.071	.74	.270	1.30	.403	1.86	.469	2.42	.492	2.98	.499
.19	.075	.75	.273	1.31	.405	1.87	.469	2.43	.493	2.99	.499
.20	.079	.76	.276	1.32	.407	1.88	.470	2.44	.493	3.00	.499
.21	.083	.77	.279	1.33	.408	1.89	.471	2.45	.493	3.01	.499
.22	.087	.78	.282	1.34	.410	1.90	.471	2.46	.493	3.02	.499
.23	.091	.79	.285	1.35	.412	1.91	.472	2.47	.493	3.03	.499
.24	.095	.80	.288	1.36	.413	1.92	.473	2.48	.493	3.04	.499
.25	.099	.81	.291	1.37	.415	1.93	.473	2.49	.494	3.05	.499
.26	.103	.82	.294	1.38	.416	1.94	.474	2.50	.494	3.06	.499
.27	.106	.83	.297	1.39	.418	1.95	.474	2.51	.494	3.07	.499
.28	.110	.84	.300	1.40	.419	1.96	.475	2.52	.494	3.08	.499
.29	.114	.85	.302	1.41	.421	1.97	.476	2.53	.494	3.09	.499
.30	.118	.86	.305	1.42	.422	1.98	.476	2.54	.495	3.10	.499
.31	.122	.87	.308	1.43	.424	1.99	.477	2.55	.495	3.11	.499
.32	.126	.88	.311	1.44	.425	2.00	.477	2.56	.495	3.12	.499
.33	.129	.89	.313	1.45	.427	2.01	.478	2.57	.495	3.13	.499
.34	.133	.90	.316	1.46	.428	2.02	.478	2.58	.495	3.14	.499
.35	.137	.91	.319	1.47	.429	2.03	.479	2.59	.495	3.15	.499
.36	.141	.92	.321	1.48	.431	2.04	.479	2.60	.495	3.16	.499
.37	.144	.93	.324	1.49	.432	2.05	.480	2.61	.496	3.17	.499
.38	.148	.94	.326	1.50	.433	2.06	.480	2.62	.496	3.18	.499
.39	.152	.95	.329	1.51	.435	2.07	.481	2.63	.496	3.19	.499
.40	.155	.96	.332	1.52	.436	2.08	.481	2.64	.496	3.20	.499
.41	.159	.97	.334	1.53	.437	2.09	.482	2.65	.496	3.21	.499
.42	.163	.98	.337	1.54	.438	2.10	.482	2.66	.496	3.22	.499
.43	.166	.99	.339	1.55	.439	2.11	.483	2.67	.496	3.23	.499
.44	.170	1.00	.341	1.56	.441	2.12	.483	2.68	.496	3.24	.499
.45	.174	1.01	.344	1.57	.442	2.13	.483	2.69	.496	3.25	.499
.46	.177	1.02	.346	1.58	.443	2.14	.484	2.70	.497	3.26	.499
.47	.181	1.03	.349	1.59	.444	2.15	.484	2.71	.497	3.27	.500
.48	.184	1.04	.351	1.60	.445	2.16	.485	2.72	.497	3.28	.500
.49	.188	1.05	.353	1.61	.446	2.17	.485	2.73	.497	3.29	.500
.50	.192	1.06	.355	1.62	.447	2.18	.485	2.74	.497	3.30	.500
.51	.195	1.07	.358	1.63	.449	2.19	.486	2.75	.497	3.31	.500
.52	.199	1.08	.360	1.64	.450	2.20	.486	2.76	.497	3.32	.500
.53	.202	1.09	.362	1.65	.451	2.21	.487	2.77	.497	3.33	.500
.54	.205	1.10	.364	1.66	.452	2.22	.487	2.78	.497		
.55	.209	1.11	.367	1.67	.453	2.23	.487	2.79	.497		

TABLE 14.15 Standard normal distribution.