Final Exam Math 1300 Spring 2012

*Instructions:* Solve 14 of the problems 1–16. If you solve more than 14 problems, you must clearly mark which 14 you want to have graded. For full credit, you must show complete, correct, legible work. Read carefully before you start working. No books or notes are allowed. Calculators are allowed, phones and PDAs are not.

1. Is the following argument valid? Use Euler circles to justify your answer.

Most of the wine produced in the U.S. is consumed in the U.S., therefore most of the wine consumed in the U.S. is domestic.

- 2. Negate the following statements.
  - (a) There exists an integer x such that  $x + x + x \ge x^3$ .
  - (b) All Canadians speak French without an accent.
  - (c) It is dry and cold today.
- 3. Consider the following figure with a diameter of 5 ft:



- (a) Find the perimeter.
- (b) Find the area of the shaded region.
- (c) Imagine that the figure is the base of a 3-dimensional object, a cylinder, that stands 2 ft tall. Determine the volume of this 3-D figure.
- 4. A game of Humans vs. Zombies starts with 400 humans. If the attrition rate is 26% per day, how many humans are left after four days of playing?
- 5. On a college campus, a survey of 70 people as to what pets they owned as children was conducted. The survey yielded the following data: 52 had a dog, 50 had a cat, 30 had a bird, 17 had a cat and a bird, 35 had a dog and a cat, 25 had a dog and a bird, and 15 had all three.
  - (a) Draw a Venn diagram that accurately depicts the data provided above.
  - (b) How many students had only a dog or only a cat, growing up?

- 6. Construct Pascals triangle and use it to answer the following questions.
  - (a) In how many ways can a person hang 5 shirts and 2 sweaters on a clothes line?
  - (b) In how many ways can a person arrange 4 blue flowers and 3 red flowers in a row?
- 7. Suppose you would like to save \$100,000 for a down payment on a home.
  - (a) In order to reach your goal in 20 years, how much should you deposit now if you make a one-time deposit into an account that bears 7.8% interest per year compounded continuously.
  - (b) If you make a one-time deposit \$550 into an account that bears 7.8% interest per year compounded continuously, how long will it take to reach your goal?
- 8. Consider the following preference lists for eight voters

| Rank   |   |   |   |   |   |   |   |   |
|--------|---|---|---|---|---|---|---|---|
| First  | C | D | B | C | A | A | C | C |
| Second | A | B | D | D | B | D | B | B |
| Third  | D | A | A | B | C | C | D | D |
| Fourth | B | C | C | A | D | B | A | A |

Determine the winner (A, B, C, or D) using

- (a) Plurality voting
- (b) Borda count
- (c) The Hare method
- (d) Pairwise comparison
- 9. Consider the following network,



- (a) Draw the minimum spanning tree.
- (b) Find the weight of the minimum spanning tree.
- 10. Build a truth table to determine when the following statement is true.

 $(p \lor q) \to (\sim p \land q)$ 

11. Consider a square with diagonal 5 cm long.



- (a) Find the perimeter.
- (b) Find the area.
- (c) Imagine that the figure above is the base of a 3-dimensional object, a box, that stands 5 cm tall. Determine the volume of the object.
- 12. In the ten years from the 2000 census to the 2010 census the U.S. population grew by 9.7% to a total of 308,745,538.
  - (a) Find the approximate size of the U.S. population in 2000.
  - (b) For the upcoming elections, the number of seats in the House of Representatives has been reapportioned according to the 2010 census data. There are 435 seats in the House; which standard divisor was used for the reapportionment?
  - (c) The table shows population data for 20 states, including all that gained or lost seats in the reapportionment. Did the population paradox occur? Explain why/why not.

|                |                  | Population       |                 |          | Seats  |
|----------------|------------------|------------------|-----------------|----------|--------|
|                | 2000             | 2010             | Change          | % change | change |
| Arizona        | 5,130,632        | 6,392,017        | 1,261,385       | 24.6     | +1     |
| Florida        | $15,\!982,\!378$ | 18,801,310       | $2,\!818,\!932$ | 17.6     | +2     |
| Georgia        | $8,\!186,\!453$  | $9,\!687,\!653$  | $1,\!501,\!200$ | 18.3     | +1     |
| Illinois       | 12,419,293       | 12,830,632       | 411,339         | 3.3      | -1     |
| Iowa           | $2,\!926,\!324$  | 3,046,355        | 120,031         | 4.1      | -1     |
| Louisiana      | 4,468,976        | 4,533,372        | 64,396          | 1.4      | -1     |
| Massachusetts  | $6,\!349,\!097$  | $6,\!547,\!629$  | $198,\!532$     | 3.1      | -1     |
| Michigan       | 9,938,444        | 9,883,640        | -54,804         | -0.6     | -1     |
| Missouri       | $5,\!595,\!211$  | $5,\!988,\!927$  | 393,716         | 7.0      | -1     |
| Ohio           | 11,353,140       | $11,\!536,\!504$ | 183,364         | 1.6      | -2     |
| Nevada         | $1,\!998,\!257$  | 2,700,551        | 702,294         | 35.1     | +1     |
| New Jersey     | 8,414,350        | 8,791,894        | $377,\!544$     | 4.5      | -1     |
| New York       | $18,\!976,\!457$ | $19,\!378,\!102$ | $401,\!645$     | 2.1      | -2     |
| North Carolina | 8,049,313        | $9,\!535,\!483$  | $1,\!486,\!170$ | 18.5     | 0      |
| Pennsylvania   | $12,\!281,\!054$ | 12,702,379       | 421,325         | 3.4      | -1     |
| South Carolina | 4,012,012        | 4,625,364        | $613,\!352$     | 15.3     | +1     |
| Texas          | 20,851,820       | $25,\!145,\!561$ | $4,\!293,\!741$ | 20.6     | +4     |
| Utah           | $2,\!233,\!169$  | 2,763,885        | 530,716         | 23.8     | +1     |
| Washington     | $5,\!894,\!121$  | 6,724,540        | 830,419         | 14.1     | +1     |
| Wisconsin      | $5,\!363,\!675$  | $5,\!686,\!986$  | $323,\!311$     | 6.0      | 0      |

13. Fill in the blanks in the sequence

$$\underline{\phantom{a}}, 8, \underline{\phantom{a}}, 27, \underline{\phantom{a}}, \cdots$$

in such a way that it becomes

- (a) an arithmetic sequence.
- (b) a geometric sequence.
- (c) a Fibonacci sequence.
- 14. Given

$$\begin{split} U &= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14\}, \\ A &= \{1, 2, 3, 4, 7, 8, 12\}, \\ B &= \{3, 6, 9, 11, 12\}, \quad and \\ C &= \emptyset \end{split}$$

find

- (a)  $|A \cap B|$ .
- (b)  $(A \cup C) \cap B$ .
- (c)  $(A \cap B) \cup C$ .
- 15. Consider the following network,



- (a) Is there an Euler circuit?
- (b) Is there a Hamiltonian cycle?
- 16. Simplify the following completely.

$$\log(3x) + 2\log(2) - 2\ln(4x) = \ln(e^x) - 2\ln(y) - \log(x) + \log(3x)$$