TOPOLOGY PRELIMINARY EXAM
AUGUST 2013

Endeavor to provide details for any arguments made during the course of a proof that are as complete as possible. If any major theorem is used in any argument, give a precise statement of that theorem. Do all 8 problems.

(1) Prove that every compact subspace of a Hausdorff topological space is closed.
(2) Compute, carefully and with all necessary details, the homology of a point.
(3) Prove that a topological space $X$ is regular if and only if for every point $x \in X$ and every open neighborhood $U$ of $X$, there is an open neighborhood $V$ of $x$ such that $V \subset U$.
(4) Suppose that a group $G$ acts properly discontinuously on a connected, locally path connected space $X$. Prove that the quotient map
$$\rho : X \to X/G$$
satisfies the axioms for a covering space.
(5) Give a statement and proof of Urysohn’s Lemma.
(6) Suppose that $f, g : X \to Y$ are two homotopic continuous maps from a space $X$ to a space $Y$. Prove that they induce the same map in homology.
(7) Compute the fundamental group of the Klein bottle.
(8) Suppose that
$$\rho : (\tilde{X}, \tilde{x}_0) \to (X, x_0)$$
is a covering space. Show that every path $\gamma : [0, 1] \to X$ with initial point $x_0$ has a unique lift $\tilde{\gamma} : [0, 1] \to \tilde{X}$ with initial point $\tilde{x}_0$. 