1. Prove that the closure of a connected set is connected.
2. Prove that a topological space satisfies the fourth separation axiom (axiom $T_4$) if and only if any neighborhood of any closed set contains the closure of some neighborhood of the same set.
3. Prove that for a metric space separability is equivalent to being second countable.
4. Prove the Lindelöf Theorem: Every open cover of a second countable space contains a countable subcover.
5. Prove that the cube $I^n$ is compact.
6. Given manifold $X = (-4, 4)$ find its class $C^r$ and state orientability if the atlas of $X$ consists of three charts \((U_1 = (-4, 0), \varphi_1(x) = 1 - x), (U_2 = (-2, 2), \varphi_2(x) = x^3), (U_3 = (0, 4), \varphi_3(x) = -x - 1)\). If $X$ happens to be orientable then construct an orientation $\omega$.
7. Compute $X$-polynomials for the link \(\infty\).
8. Compute: (i) the fundamental group $\pi_1(X, x_0)$ of the space $X$ (see Fig. 1), (ii) the fundamental group $\pi_1(X \times Y)$ where $X$ and $Y$ are depicted in Figs 1 and 2, and (iii) the fundamental group $\pi_1(X \vee Y)$ where $X$ and $Y$ are depicted in Figs 1 and 2.

![Fig. 1.](image1)
![Fig. 2.](image2)

9. The generators $g_1$ and $g_2$ of the fundamental group of a topological space $X$ homeomorphic to a disk with two holes are depicted in Fig. 3. Compute the homotopy class of the loop $\alpha$ depicted in Fig. 4.

![Fig. 3.](image3)
![Fig. 4.](image4)

10. Compute the fundamental group of the space $X$ which is homeomorphic to a projective plane with three holes.